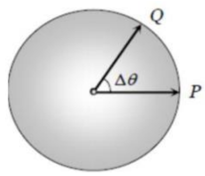
UNIT 6

ROTATIONAL MOTION

6.1 Angular Displacement

It is the angle described by the position vector r about the axis of rotation



Angular displacement $(\theta)=\frac{Linear\ displacement\ (s)}{Radius\ (r)}$

It is dimensionless quantity, Vector form: $\vec{s}=\vec{\theta}\times\vec{s}$

i.e., angular displacement is a vector quantity whose direction is given by right hand rule. It is also known as axial vector. For anti-clockwise sense of rotation, direction of **s** is perpendicular to the plane, outward and along the axis of rotation and vice-versa.

Angular Velocity ($\vec{\omega}$):

The angular displacement per unit time is defined as angular velocity.

If a particle moves from P to Q in time Δt, $\vec{\omega}=\frac{\Delta\vec{\theta}}{\Delta t}$ where $\Delta\vec{\theta}$ is the angular displacement.

Instantaneous angular velocity $\vec{\omega}=\lim_{\Delta t\to 0}\frac{\Delta\vec{\theta}}{\Delta t}=\frac{d\vec{\theta}}{dt}$

Average angular velocity $\vec{\omega}_{av}=\frac{total\ angular\ displacement}{total\ time}=\frac{\theta_2-\theta_1}{t_2-t_1}$

If the position vector of a particle is $\vec{r}=(3i+4j)$ meter and its angular velocity is $\vec{\omega}=(j+2k)$ in rad/sec then its linear velocity is (in m/s)

Solution:

$$\vec{v}=\vec{\omega}\times\vec{r}=\begin{vmatrix} i & j & k \\ 0 & 1 & 2 \\ 2 & 4 & 0 \end{vmatrix}=-8i\quad+4j\quad-2k$$

6.3: Angular Acceleration

The rate of change of angular velocity is defined as angular acceleration

If particle has angular velocity $\vec{\omega}_1$ at time t_1 and angular velocity $\vec{\omega}_2$ at time t_2 then,

Angular acceleration: $\vec{\alpha}=\frac{\vec{\omega}_2-\vec{\omega}_1}{t_2-t_1}$

Instantaneous angular acceleration $\vec{\alpha}=\lim_{\Delta t\to 0}\frac{\Delta\vec{\omega}}{\Delta t}=\frac{d\vec{\omega}}{dt}=\frac{d^2\vec{\theta}}{dt^2}$

If $\vec{\alpha} = \mathbf{0}$, circular or rotational motion is said to be uniform.

Average Angular acceleration: $\vec{\alpha}_{av} = \frac{\vec{\omega}_2 - \vec{\omega}_1}{t_2 - t_1}$

Relation between angular acceleration and linear acceleration: $\vec{a} = \vec{\alpha} \times \vec{r}$ (vector form)

It is an axial vector whose direction is along the change in direction of angular velocity *i.e.* normal to the rotational plane, outward or inward along the axis of rotation (depends upon the sense of rotation).

Example.

1. The wheel of a car is rotating at the rate of 1200 revolutions per minute. On pressing the accelerator for 10 sec it starts rotating at 4500 revolutions per minute. Find the angular acceleration of the wheel.

Given:

ω_o =1400rev/min

ω =5,400rev/min

t=10sec

Req[d]

α =?

Solution:

$\alpha = \frac{\omega - \omega_o}{t}$

1rev/min= $\frac{2\pi rad}{60sec}$

ω_o= 1400rev/min

=1,400*$\frac{2\pi rad}{60sec}$=**146.53rad/se**

ω= 5400rev/min

=5,400*$\frac{2\pi rad}{60sec}$=**565.2rad/se**

$\alpha = \frac{(565.2 - 146.53)\text{rad/sec}}{10sec}$

$\alpha = 41.87\ ^{rad}/_{sec^2}$

2. A wheel is at rest. Its angular velocity increases uniformly and becomes 60rad/sec after 5sec. The total angular displacement is:

Solution:

Angular acceleration=$\alpha = \frac{\omega - \omega_o}{t} = \frac{60rad/sec - 0rad/sec}{5sec.}$=$\mathbf{12}\ ^{\boldsymbol{rad}}/_{\boldsymbol{sec^2}}$

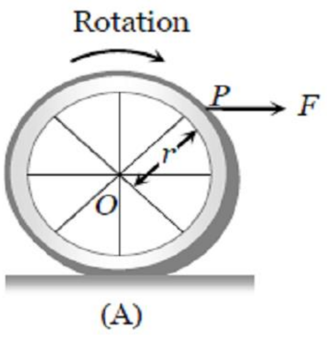
Angular displacement=$\theta = 0 + \frac{1}{2}(12\ ^{rad}/_{sec^2})(5sec)^2$=**150rad**

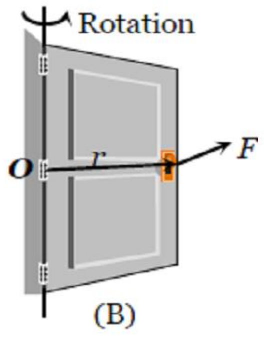
Rotation of a rigid body about a fixed axis

Axis of rotation the axis about which a body rotates

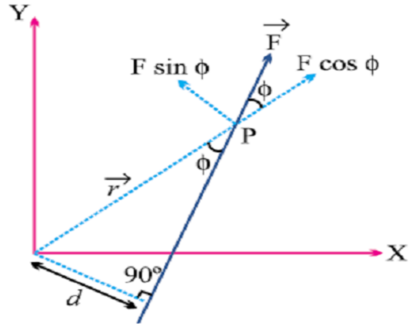
Torque and angular acceleration

Torque is the turning effect of force round a point.





If the particle rotating in xy plane about the origin under the effect of force $\vec{\boldsymbol{F}}$ and at any instant the position vector $\vec{\boldsymbol{r}}$ of the particle is then,



Torque $\vec{\tau} = \vec{\boldsymbol{r}} \times \vec{F}$

$\vec{\tau} = |\vec{\boldsymbol{r}}||\vec{F}|sin\theta\hat{u}$

Where θ is the angle between the direction of $\vec{\boldsymbol{r}}$ *and* $\vec{F}$

Torque is an axial vector *i.e.,* its direction is always perpendicular to the plane containing vector $\vec{\boldsymbol{r}}$ *and* $\vec{F}$ in accordance with right hand screw rule. For a given figure the sense of rotation is anti-clockwise so the direction of torque is perpendicular to the plane, outward through the axis of rotation.

A body is said to be in rotational equilibrium if resultant torque acting on it is zero *i.e.,* $\sum \vec{\tau} = 0$

Torque is the cause of rotatory motion and in rotational motion it plays same role as force plays in translational motion *i.e.,* torque is rotational analogue of force.

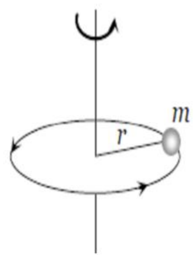
1. Find the torque of 7i-3j-5k about the origin which acts on a particle whose position vector is i+j-k.

$$\text{Solution: } \vec{\tau} = \vec{\boldsymbol{r}} \times \vec{F} = \begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ 7 & -3 & -5 \end{vmatrix} = -8i \quad -2j \quad -10k$$

6.4: **Rotational kinetic energy and rotational inertia.**

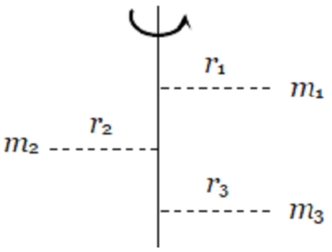
Rotational inertia: is a measure of an object's resistance to changes in its speed of rotation over a certain time. It is also known as moment of inertia. It plays the same role in rotational motion as mass plays in linear motion.

1. Moment of inertia of a point mass at r distance from axis of rotation.



$I = mr^2$

2. Moment of inertia of a body made up of number of particles (discrete distribution) is given by $\mathrm{I} = \mathrm{m_1 r_1}^2 .+ \mathrm{m_2 r_2}^2 + \cdots + \mathrm{m}_n \mathrm{r}_n{}^2 = \sum_{i=1}^{n} \mathrm{m}_n \mathrm{r}_n{}^2$



$$\mathbf{I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2}$$

Moment of inertia depends on mass, distribution of mass and on the position of axis of rotation. Moment of inertia does not depend on angular velocity, angular acceleration, torque, angular momentum and rotational kinetic energy.

Example:

1. Five particles of mass = 4kg are attached to the rim of a circular disc of radius 0.2m and negligible mass. Find the moment of inertia of the system about the axis passing through the centre of the disc and perpendicular to its plane.

Given:

$m_1 = m_2 = m_3 = m_4 = m_5 = 4\text{kg}$

$\mathbf{r_1 = r_2 = r_3 = r_4 = r_5 = r = 0.2m}$

Required: I=?

solution:

$$\mathbf{I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2 + m_5 r_5^2}$$

$$\mathbf{= 5mr^2 = 5(4kg)(0.2)^2}$$

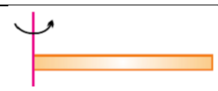
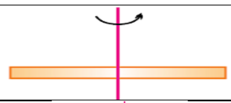
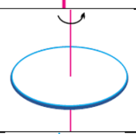
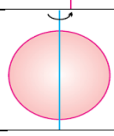
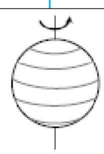
$$\mathbf{I = 0.8kgm^2}$$

Moments of inertia of different rigid bodies:

Different rigid bodies have different moment of inertia depending on size mass, distribution of mass, shape of the body and on the position of axis of rotation.

Moment of Inertia of Some Standard Bodies about different Axes:

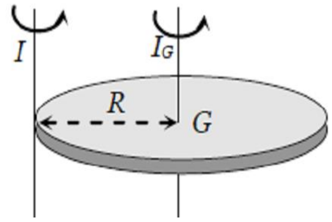
Parallel axis theorem

Body	Axis of Rotation	Figure	Moment of inertia
Long thin rod	About an axis passing through its edge and perpendicular to the rod		$I = \frac{ML^2}{3}$
Long thin rod	About on axis passing through its centEr of mass and perpendicular to the rod.		$I = \frac{ML^2}{12}$
Disc	About an axis passing through C.G. and perpendicular to its plane		$I = \frac{Mr^2}{2}$
Disc	About its Diameter		$I = \frac{Mr^2}{4}$
solid sphere	About its Diameter		$I = \frac{2}{5}Mr^2$

Parallel axis theorem

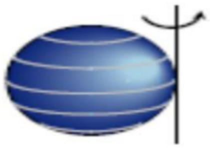
Moment of inertia of a body about a given axis I is equal to the sum of moment of inertia of the body about an axis parallel to given axis and passing through center of mass of the body $\mathbf{I}_g$ and $\mathbf{MR^2}$ where **M** is the mass of the body and **R** is the perpendicular distance between the two axes..

$$I = \mathrm{I}_g + \mathrm{MR}^2$$



Example.

1. **Moment of inertia of a solid sphere about an axis through its centre is 2/5MR², Find moment of inertia of solid sphere about it tangential axis.**

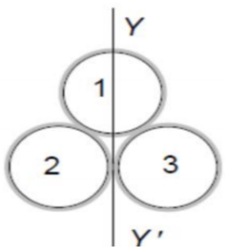


Solution:

$$I = \mathrm{I}_g + \mathrm{MR}^2$$

$$I = \frac{2}{5}\mathrm{MR}^2 + \mathrm{MR}^2 = \frac{7}{5}\mathrm{MR}^2$$

2. **Three discs each of mass M and radius R are arranged as shown in the figure.**
   Find the moment of inertia of the system about YY' I = 1/4(MR²)



Solution:

$$\mathbf{I} = \mathrm{I}_1 + \mathrm{I}_2 + \mathrm{I}_3$$

I_1 is about its diameter and, I_2 and I_3 are about tangential axis

$$I = \mathrm{I}_1 + \mathrm{I}_2 + \mathrm{I}_3$$

$$\mathrm{I}_1 = \frac{1}{4}MR^2 \text{ and}$$

$$\mathrm{I}_1 + \mathrm{I}_2 + \mathrm{I}_3 = \frac{1}{4}MR^2 + 2\left(\frac{1}{4}\mathrm{MR}^2 + \mathrm{MR}^2\right)$$

$$\mathrm{I} = \frac{11}{4}\mathrm{MR}^2$$

Rotational Work Done

Rotational work done by torque is the scalar product of moment of force and angular displacement. It is given by: $W = \tau\theta$

Torque in terms moment of inertia and angular acceleration

Torque(τ) is the vector product of lever arm and the force applied. It is given by:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = \mathrm{rF}\sin\theta,\ if\ \theta = 90^\circ\ \sin\theta = 1, \text{ then}$$

$$\tau = \mathrm{rF} = \mathrm{r}(ma_t) = rm\alpha r = (mr^2)\alpha = I\alpha,$$

where **τ-torque, I-moment of inertia and α-angular acceleration**

Rotational kinetic energy

Rotational kinetic energy the amount of kinetic energy a rigid body has from its rotational movement. A body rotating about a fixed axis possesses kinetic energy because its constituent particles are in motion, even though the body as a whole remains in place. It is given by:

$KE_{rotational}$ = 1/2 I^2

Rotational kinetic energy	Analogue to translational kinetic energy
$KE_r = \frac{1}{2} I\omega^2$	$KE_r = \frac{1}{2} mv^2$
$KE_r = \frac{1}{2} L\ \omega$	$KE_r = \frac{1}{2}\mathbf{p}v$
$KE_r = \frac{L^2}{2I}$	$KE_r = \frac{P^2}{2m}$

The angular velocity of a body is $\vec{\omega} = (\hat{\imath} + 10\hat{\jmath} + 4\hat{k})$rad/sec and a torque $\vec{\boldsymbol{\tau}} = (\hat{\imath} - 4\hat{\jmath} + 4\hat{k})$Nm acts on it. Find the rotational power delivered.

Given:

$\vec{\omega} = (\hat{\imath} + 10\hat{\jmath} + 4\hat{k})$rad/sec

$\vec{\boldsymbol{\tau}} = (\hat{\imath} - 4\hat{\jmath} + 4\hat{k})$Nm

Required:

P =?

solution:

$$\mathbf{P} = \vec{\boldsymbol{\tau}}.\vec{\omega}$$

$$\mathbf{P} = (\hat{\imath} - 4\hat{\jmath} + 4\hat{k}).\ (\hat{\imath} + 10\hat{\jmath} + 4\hat{k})$$

$$\mathbf{P} = (1)(1) + (-4)(10) + (4)(4)$$

$$\mathbf{P} = \mathbf{55}\boldsymbol{Watt}$$

Rolling Without Slipping

In case of combined translational and rotary motion if the object rolls across a surface in such a way that there is no relative motion of object and surface at the point of contact, the motion is called rolling without slipping. Friction is responsible for this type of motion but work done or dissipation of energy against friction is zero as there is no relative motion between body and surface at the point of contact. Rolling motion of a body may be treated as a pure rotation about an axis through point of contact with same angular velocity . By the law of conservation of energy

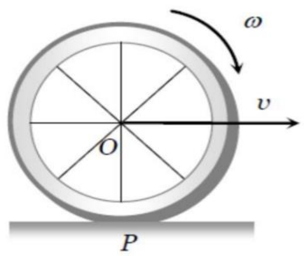
$$KE_{Total} = KE_{rot} + KE_{tran}$$

$$KE_{Total} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$\boldsymbol{v} = \omega\boldsymbol{r}$$

$$KE_{Total} = \frac{1}{2}m\omega^2 r^2 + \frac{1}{2}I\omega^2$$

$$KE_{Total} = \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{\omega}^{\mathbf{2}}(\boldsymbol{mr}^{\mathbf{2}} + \mathbf{I})$$



Rolling Down on Smooth Inclined Plane without Slipping

When a body of mass m and radius r rolling down on inclined plane without slipping when released from ‘h’ and angle of inclination with the horizontal is θ. It loses its potential energy. However it acquires both linear and angular speeds and hence, gains kinetic energy of translational and that of rotation.

Example:

A ball of mass m and radius r is released from 'h' on a smooth inclined plane of inclination as shown below. Find it's a) linear speed in terms of 'g' and 'h' b) its angular speed in terms of 'g', 'h' and radius 'r', and c) linear acceleration in terms of 'g' and ' θ '.

Solution:

If friction is zero, from the la of conservation of mechanical energy:

$$\mathrm{ME_{top}} = \mathrm{ME_{bottom}}$$

$$\mathrm{GPE_{top}} = \mathrm{KE_{Rtranslational\ at\ bottom}} + \mathrm{KE_{Rotational\ at\ bottom}}$$

$$\mathrm{mgh} = \frac{1}{2}mv^2 + \frac{1}{2}\mathrm{I_{ball}}\omega^2,$$

$$\mathrm{I_{ball}} = \mathrm{I_{hollow\ sphere}} = \frac{2}{3}mr^2 \text{ and } \boldsymbol{v} = \omega r$$

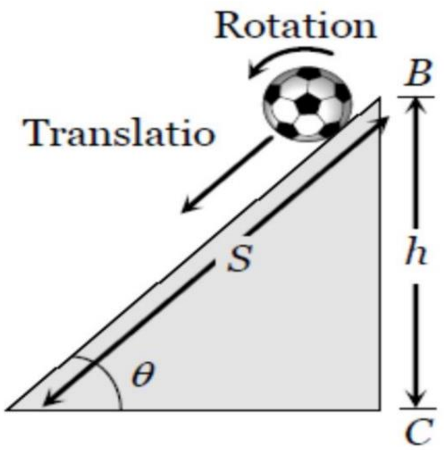
$$\Rightarrow \mathrm{mgh} = \frac{1}{2}mv^2 + \frac{1}{2}(\frac{2}{3}mr^2)\omega^2$$

$$\Rightarrow \mathrm{mgh} = \frac{1}{2}mv^2 + \frac{1}{3}m(\omega r)^2$$

$$\Rightarrow \mathrm{mgh} = \left(\frac{1}{2} + \frac{1}{3}\right)mv^2 = \frac{5}{6}mv^2$$

$$\mathrm{gh} = \frac{5}{6}v^2$$

$$\boldsymbol{v} = \sqrt{\frac{6}{5}\boldsymbol{gh}}, \quad \& \quad \boldsymbol{\omega} = \frac{1}{r}\sqrt{\frac{6}{5}\boldsymbol{gh}}$$



Angular momentum and angular impulse

The turning momentum of particle about the axis of rotation is called the angular momentum of the particle or the moment of linear momentum of a body with respect to any axis of rotation is known as angular momentum. If P is the linear momentum of particle, and r its position vector from the point of rotation then angular momentum.

$$L_z = \sum_i L_i = \sum_i m_i r_i^2 \omega = \left(\sum_i m_i r_i^2\right)\omega$$

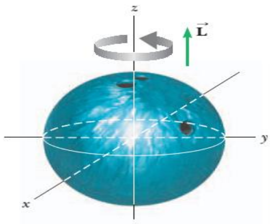
$$L_z = I\omega$$

and

$$\sum \tau_{\text{ext}} = I\alpha$$

If a symmetrical object rotates about a fixed axis passing through its centre of mass, you can write the above equation in vector form as, **L = I** where **L** is the total angular momentum of the object measured with respect to the axis of rotation. Furthermore, the expression is valid for any object, regardless of its symmetry, if **L** stands for the component of angular momentum along the axis of rotation**.**

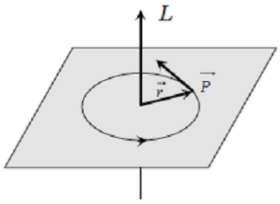
Estimate the magnitude of the angular momentum of a bowling ball spinning at 10 rev/s as shown in Figure



$$I = \tfrac{2}{5}MR^2 = \tfrac{2}{5}(7.0\ \text{kg})(0.12\ \text{m})^2 = 0.040\ \text{kg}\cdot\text{m}^2$$

$$L_z = I\omega = (0.040\ \text{kg}\cdot\text{m}^2)(10\ \text{rev/s})(2\pi\ \text{rad/rev}) = 2.53\ \text{kg}\cdot\text{m}^2/\text{s}$$

In vector form

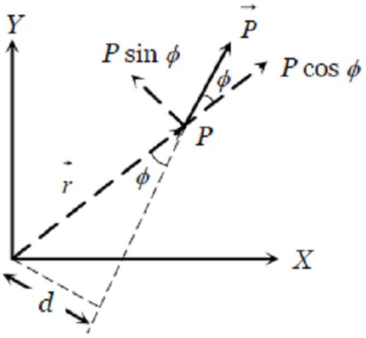


$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{P}}$

$\vec{\mathbf{L}} = rpsin\theta\hat{\mathbf{n}}$

Angular momentum is an axial vector *i.e.* always directed perpendicular to the plane of rotation and along the axis of rotation.

S.I. Unit of $\vec{\mathbf{L}}$: kgm^2s^{-1} *or* J.sec.



In case of circular motion angular momentum is given by:

$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{P}} = m(\vec{\mathbf{r}} \times \vec{v}) = mvrsin\emptyset\hat{\mathbf{n}}$,

if $\emptyset$ between is 90°, then

$L = mvr = m(\omega r)r = I\omega$

In vector form $\vec{\mathbf{L}} = I\vec{\omega}$, $I = mr^2$

In Cartesian co-ordinates if $\vec{\mathbf{r}} = xi + y$

Then $\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{P}} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \\ x & y & y \\ P_x & P_y & P_z \end{vmatrix}$ =$(y\mathbf{P}_z - z\mathbf{P}_y)\hat{\imath}$ - $(\mathbf{x}\mathbf{P}_z - z\mathbf{P}_x)\hat{\jmath}$ + $(x\mathbf{P}_y - y\mathbf{P}_x)\hat{k}$

Worked Example: The position of a particle is given by : $\vec{\mathbf{r}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and momentum $\vec{\mathbf{P}} = \mathbf{2}\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$ .Find the direction of angular momentum

Solution:

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{P}} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x & y & y \\ P_x & P_y & P_z \end{vmatrix}$$

Direction of $\vec{\mathbf{L}}$ is its unit vector given by: $\hat{\boldsymbol{u}}=\frac{\vec{\mathbf{L}}}{|\vec{\mathbf{L}}|} = \frac{\vec{\mathbf{L}}}{|\vec{\mathbf{L}}|}$

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{P}} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & 1 \\ 0 & 2 & 3 \end{vmatrix} = \hat{\mathbf{i}} - 3\,\hat{\mathbf{j}} + 2\hat{\mathbf{k}}$$

$$\hat{\boldsymbol{u}}=\frac{\vec{\mathbf{L}}}{|\vec{\mathbf{L}}|} = \frac{\hat{\mathbf{i}} - 3\,\hat{\mathbf{j}} + 2\hat{\mathbf{k}}}{\sqrt{1^2+(-3)^2+2^2}} = \frac{\hat{\mathbf{i}} - 3\,\hat{\mathbf{j}} + 2\hat{\mathbf{k}}}{\sqrt{14}} \text{ (unit vector of } \vec{\mathbf{L}})$$

Angular impulse

Angular impulse the change in angular momentum of a rotating body caused by a torque acting over a certain time. It is given by:

$$\vec{\mathbf{J}} = \Delta\vec{\mathbf{L}} = \vec{\tau}_{av}\Delta t$$

The angular momentum of a system of particles is equal to the vector sum of angular momentum of each particle *i.e.*, $\vec{\mathbf{L}} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3 + \cdots + \vec{L}_n$

Analogy Between Translational Motion and Rotational Motion

Translational Motion		**Rotational Motion**	
Mass (m)		Moment of Inertia (I)	
Linear momentum	$\vec{\mathbf{P}} = \boldsymbol{m}\vec{\boldsymbol{v}}$ $\mathbf{P} = \sqrt{\mathbf{2}\boldsymbol{mE}}$	Angular Momentum	$L = I\omega$ $L = \sqrt{\mathbf{2}\boldsymbol{IE}}$
Force	**F=ma**	**Torque**	$\boldsymbol{\tau}$ =I$\boldsymbol{\alpha}$
Kinetic Energy	$\mathbf{KE}_t=\frac{\mathbf{1}}{\mathbf{2}}\mathbf{I}\boldsymbol{v}^{\mathbf{2}}$ $\mathbf{KE}_t=\frac{\mathbf{P^2}}{\mathbf{2m}}$	Rotational Kinetic Energy	KE $=\frac{1}{2}\mathbf{I}\boldsymbol{\omega}^{\mathbf{2}}$ $\mathbf{KE}_R=\frac{\mathbf{L^2}}{\mathbf{2I}}$

The Law of Conservation of angular momentum

Newton's second law for rotational motion $\quad \vec{\boldsymbol{\tau}} = \frac{d\vec{\mathbf{L}}}{dt}$

So if the net external torque on a particle (or system) is zero then $\frac{d\vec{\mathbf{L}}}{dt}$=0, *i.e.* $\vec{\mathbf{L}} = \vec{L}_1 + \vec{L}_2 + \vec{L}_3 + \cdots + \vec{L}_n$=*constant*

Angular momentum of a system (may be particle or body) remains constant if resultant torque acting on it zero.

As form $\vec{\mathbf{L}} = I\vec{\omega}$ so if $\vec{\boldsymbol{\tau}} = 0$ then $I\omega = constant$ $\quad \mathbf{I}\sim\frac{\mathbf{1}}{\boldsymbol{\omega}}$

Since angular momentum $\mathbf{I\omega}$ remains constant so when $\mathbf{I}$ decreases, angular velocity $\boldsymbol{\omega}$ increases and vice-versa.

Example:

1. **Two discs of moment of inertia 5kg and 10kgm and angular speeds 12rad/sec and 4rad/sec are rotating along collinear axes passing through their centre of mass and perpendicular to their plane. If the two are made to rotate together along the same axis, then what is rotational KE of system after they made to rotate together?**

Solution:

$$\mathbf{L}_o = \mathbf{L}_f$$

$$\mathbf{I}_{1i}\boldsymbol{\omega}_{1i} + \mathbf{I}_{2i}\boldsymbol{\omega}_{2i} = \mathbf{I}_{1f}\boldsymbol{\omega}_{1f} + \mathbf{I}_{2f}\boldsymbol{\omega}_{2f}$$

$$\mathbf{I}_{1i}\boldsymbol{\omega}_{1i} + \mathbf{I}_{2i}\boldsymbol{\omega}_{2i} = (\mathbf{I}_{1f} + \mathbf{I}_{2f})\boldsymbol{\omega}_f$$

$$\boldsymbol{\omega}_f = \left(\frac{\boldsymbol{\omega}_{1i}\mathbf{I}_1 + \mathbf{I}_2\boldsymbol{\omega}_{2i}}{\mathbf{I}_1 + \mathbf{I}_2}\right) = \frac{12rad/sec * \mathbf{5kg}\boldsymbol{m}^2 + 4rad/sec * \mathbf{10kg}\boldsymbol{m}^2}{\mathbf{5kg}\boldsymbol{m}^2 + \mathbf{10kg}\boldsymbol{m}^2}$$

$$\boldsymbol{\omega}_f = \mathbf{6.67rad/sec}$$

Self-Practice Questions and Problems

1. A child has a toy tied to the end of a string and whirls the toy at constant speed in a horizontal circular path of radius R. The toy completes each revolution of its motion in a time period T. What is the magnitude of the acceleration of the toy?

A. $\frac{\pi R}{T^2}$ B. $\frac{4\pi^2 R}{T^2}$ C. g D. Zero

2. A centripetal force of 5N is applied to a rubber stopper moving at a constant speed in a horizontal circle. If the same force is applied, but the radius is made smaller, what happens to the speed, v, and the frequency, f, of the stopper?

A. v increases & f increases **C. v decreases & f increases**

B. v decreases & f decreases **D. v increases & f decreases**

3. A door is free to rotate about its hinges. A force F applied normal to the door a distance x from the hinges produces an angular acceleration of α. What angular acceleration is produced if the same force is applied normal to the door at a distance of 2x from the hinges?

A. α B. 2α C. $\frac{1}{2}\alpha$ D. $\frac{1}{4}\alpha$

4. A solid disc has a rotational inertia that is equal to_____ where M is the disc's mass and R is the disc's radius. It is rolling along a horizontal surface without slipping with a linear speed of v. How are the translational kinetic energy and the rotational kinetic energy of the disc related?

A. Rotational kinetic energy is equal to the translational.

B. Translational kinetic energy is larger than rotational.

C. Rotational kinetic energy is larger than translational.

D. The answer depends on the density of the disc.

5. A child whirls a ball at the end of a rope, in a uniform circular motion. Which of the following statements is NOT true?

A. The speed of the ball is constant

B. The magnitude of the ball's acceleration is constant

C. The velocity is of the ball is constant

D. The acceleration of the ball is directed radially inwards towards the centre.

6. An automobile moves in a circle of radius 110 meters with a constant speed of 33m/s. What is the angular velocity of the car about the centre of the circle in radians per second? A. 0.3rad/s B. 0.3πrad/s C. 0.6πrad/s D. 3.3πrad/s

7. A point mass of 500g started from rest rotates on a circular path of radius 2m and rotates with a uniform angular acceleration to attain angular velocity of 4rev/sec in 2seconds. What is the moment of force acted on it to rotate with this acceleration?

A. $8\pi N.m$ B. $0.2\pi N.m$ C. $12 N.m$ D. $4.5 N.m$

8. In a uniform circular motion, the centripetal acceleration of a body moving in a circular path results from:

A. Change in magnitude of tangential velocity.

B. Change in direction of angular velocity

C. Change in direction of tangential velocity

D. Change in magnitude of tangential acceleration.

9. Which one of the following is incorrect about centripetal force?

A. As a car makes a turn, the force of friction acting upon the turned wheels of the car provides centripetal force required for circular motion.

B. As a bucket of water is tied to a string and spun in a circle, the tension force acting upon the bucket provides the centripetal force required for circular motion.

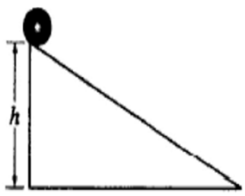
C. The centripetal force for uniform circular motion alters the direction of the object by altering its speed.

D. As the moon orbits the Earth, the force of gravity acting upon the moon provides the centripetal force required for circular motion.

10. A bowling ball of mass M and radius R. whose moment of inertia about its centre is rolls without slipping along a level surface at speed v. The maximum vertical height to which it can roll if it ascends an incline is:

A. $\frac{7v^2}{10g}$ **B.** $\frac{v^2}{2g}$ C. $\frac{v^2}{5g}$ D. $\frac{5v^2}{7g}$

11. A sphere of mass M, radius r, and rotational inertia I is released from rest at the top of an inclined plane of height h as shown below.

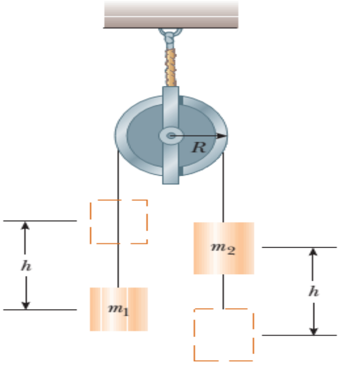


If the plane is frictionless, what is the speed v_{cm}, of the center of mass of the sphere at the bottom of the incline?

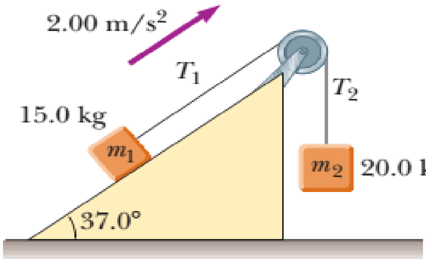
A. $\sqrt{\frac{2Mghr^2}{I}}$ B. $\sqrt{2gh}$ C. $\sqrt{\frac{2Mghr^2}{I+Mr^2}}$ D. $\frac{2Mghr^2}{I}$

Solve the following problems by showing all the necessary steps clearly.

1. Two cylinders having different masses m_1 and m_2 are connected by a string passing over a pulley, as shown in the figure below. The pulley has a radius **R** and moment of inertia **I** about its axis of rotation. The string does not slip on the pulley, and the system is released from rest. Find the translational speeds of the cylinders after cylinder 2 descends through a distance h, and find the angular speed of the pulley at this time.



2. Two blocks as shown in figure below are connected by a string of negligible mass passing over a pulley of radius 0.25 m and moment of inertia **I**. The block on the frictionless incline is moving up with a constant acceleration of 2m/s^2
   A. Determine **T_1** and **T_2**, the tensions in the two parts of the string.
   B. Find the moment of inertia of the pulley.



3. A father of mass **m_f** and his daughter of mass **m_d** sit on opposite ends of a seesaw at equal distances from the pivot at the centre as shown. The seesaw is modelled as a rigid rod of mass **M** and length **ℓ** and is pivoted without friction. At a given moment, the combination rotates in a vertical plane with an angular speed **.**
   (A) Find an expression for the magnitude of the system's angular momentum.
   (B)Find an expression for the magnitude of the angular acceleration of the system when the seesaw makes an angle θ with the horizontal.