

# Hill Side School

## *Excel with our support*

Score	%
Tests	
Final	
Others	
Total	

Central Administrative office (CMC Campus)

Primary Division phone 011-646-69 40  
High School Division Phone 011-646-36 88

Lem Campus (Lem Hotel Area)

Primary Division Phone 011-662 42 31/2

KG Campus (Kotebe Area)

Kindergarten Division Phone 011- 645 74 44

P.O.Box 21616 Addis Ababa, Ethiopia



Hillside @ ethionet.et

Full Name \_\_\_\_\_ Grade -11 Section \_\_\_\_\_ 4th Quarter Final Exam

Subject – **MATHEMATICS** for **NATURAL SCIENCE** Year: 2012 E.C Time allowed 2.5 hrs.

**ALL answers for ALL parts should be given on the separate answer sheet.**

**PART I: Choose the correct answer and write the letter of your choice in BLOCK LETTERS on the Separate answer sheet provided. (Total of 40 pts.)**

- If  $x < 0$ , then the simplest form of the expression  $\frac{x - |x|}{x}$  is equal to  
 (A)  $2x$  (B)  $2$  (C)  $-2$  (D)  $0$
- If  $f(x) = \frac{\sqrt{x+2}}{x+2}$  and  $g(x) = \frac{1}{x} - 2$ , then  $f(g(x))$  is equal to  
 (A)  $\sqrt{x} - 2$  (B)  $\sqrt{x} + 2$  (C)  $\sqrt{x}$  (D)  $\frac{\sqrt{x}}{x}$
- Which one of the following is the domain of  $f(x) = x - \frac{x^2 + 1}{x^4 + x}$ ?  
 (A)  $\mathbb{R} / \{0\}$  (B)  $\mathbb{R} / \{-1\}$  (C)  $\mathbb{R} / \{0, 1\}$  (D)  $\mathbb{R} / \{-1, 0\}$
- What are the values of  $a$  and  $b$  which make the mathematical sentence  $\frac{x+1}{x^2-9} = \frac{a}{x-3} + \frac{b}{x+3}$  true for all real numbers  $x \neq 3, -3$ ?  
 (A)  $a=1, b=0$  (B)  $a=\frac{1}{3}, b=\frac{3}{5}$  (C)  $a=-\frac{2}{5}, b=1$  (D)  $a=\frac{2}{3}, b=\frac{1}{3}$
- What is the solution set of equation  $1 - \frac{5}{x^2+4} = \frac{x-1}{x^2-x} - \frac{1}{x}$ ?  
 (A)  $\{1, -1\}$  (B)  $\{2\}$  (C)  $\{-1\}$  (D)  $\{1, 2, -1\}$
- If  $f(x) = \ln\left(\frac{x}{x-1} + 2\right)$ , then for  $x > 1$ , which one of the following is the inverse of  $f$ ?  
 (A)  $f^{-1}(x) = \frac{e^x - 2}{e^x - 3}$  (C)  $f^{-1}(x) = \frac{e^x - 2}{e^x + 1}$   
 (B)  $f^{-1}(x) = \frac{e^x}{e^x + 1} - 2$  (D)  $f^{-1}(x) = e^{\left(\frac{x}{x-1}\right)} - 2$

7. The line  $\ell$  passes through  $(0,5)$  and  $(-5,0)$ . What is the measure of the angle between the y-axis and the line  $\ell$ ?
- (A)  $\frac{\pi}{4}$                                       (B)  $\frac{\pi}{3}$                                       (C)  $\frac{\pi}{2}$                                       (D)  $\frac{3}{2}\pi$
8. Consider a circle whose center is on the x-axis. If a line given by  $y = x$  is tangent to the circle at point  $(2, 2)$ , what is the equation of the circle?
- (A)  $x^2 + y^2 = 8$                                       (C)  $(x-2)^2 + y^2 = 4$   
 (B)  $(x-4)^2 + y^2 = 8$                                       (D)  $(x-1)^2 + y^2 = 5$
9. What is the vertex and the equation of the directrix, respectively, of the parabola  $x + y^2 + 2y + 1 = 0$ ?
- (A)  $(0, -1), x = -\frac{1}{4}$                                       (C)  $(0, -1), x = \frac{1}{4}$   
 (B)  $(-1, 0), y = -\frac{1}{4}$                                       (D)  $(-1, 0), y = \frac{1}{4}$
10. Let the center of an ellipse be at  $(1,4)$  and two of its vertices be at  $(10,4)$  and  $(1,2)$ . What is the equation of the ellipse?
- (A)  $4(x-1)^2 + 81(y-4)^2 = 324$                                       (C)  $9(x-1)^2 + 4(y-4)^2 = 4$   
 (B)  $(x-1)^2 + 9(y-4)^2 = 4$                                       (D)  $2(x-1)^2 + 9(y-4)^2 = 4$
11. The equation  $x^2 - 4y^2 = -1$  represents
- (A) a hyperbola with one of its foci at the at the origin.  
 (B) a parabola with vertex at the origin.  
 (C) a hyperbola with vertices at  $\left(-\frac{1}{2}, 0\right)$  and  $\left(\frac{1}{2}, 0\right)$ .  
 (D) a hyperbola with foci at  $\left(0, -\frac{\sqrt{5}}{2}\right)$  and  $\left(0, \frac{\sqrt{5}}{2}\right)$ .
12. Which of the following is an asymptote to the hyperbola  $4x^2 - y^2 + 2y = 5$
- (A)  $y = -2x + 1$                                       (B)  $y = 2x - 1$                                       (C)  $y = -\frac{1}{2}x + 1$                                       (D)  $y = -\frac{1}{2}x - 1$
13. Let  $p(x): x^2 + x > 0$ . Which of the following is not equivalent to  $\neg(\exists x)(x^2 + x > 0)$ ?
- (A)  $(\exists x)\neg(x^2 + x > 0)$                                       (B)  $(\forall x)(x^2 + x < 0)$                                       (C)  $(\exists x)(x^2 + x \leq 0)$                                       (D)  $(\forall x)(x^2 + x \leq 0)$
14. If compound proposition  $(p \wedge q) \Rightarrow (\neg s \vee r)$  is false, which of the following is true?
- (A)  $s \Rightarrow r$                                       (B)  $\neg p \vee \neg s$                                       (C)  $(p \wedge \neg q) \Rightarrow r$                                       (D)  $p \wedge r$
15. The population  $a, b, 8, 5, 7$  has a mean of 6 and variance of 2. If  $a > b$ , then the values of  $a$  and  $b$  respectively are
- (A) 7 and 3                                      (B) 8 and 2                                      (C) 9 and 1                                      (D) 6 and 4

16. Let  $A = \begin{pmatrix} -2 & 0 & x \\ 2y & x+y & -4 \end{pmatrix}$  and  $B = \begin{pmatrix} 1 & -y \\ 0 & 3 \\ 1-x & 2 \end{pmatrix}$  such that  $A + 2B^T = 0$ . Then which one of the following is the value of  $y$ ?

- (A) 0                                      (B)  $-\frac{13}{2}$                                       (C) -8                                      (D) any real number.

17. If  $A = \begin{pmatrix} 4 & -3 \\ -1 & x \end{pmatrix}$  is an invertible matrix and  $|A^{-1}| = 1$ , then what is the value of  $x$ ?

- (A) 1                                      (B) 7                                      (C) 11                                      (D) 17

18. What is the inverse of  $A = \begin{pmatrix} 5 & -4 \\ 3 & -2 \end{pmatrix}$ ?

- (A)  $\begin{pmatrix} 1 & -2 \\ 3 & -5 \end{pmatrix}$                                       (B)  $\begin{pmatrix} -1 & 2 \\ -3 & 5 \end{pmatrix}$                                       (C)  $\begin{pmatrix} 3 & -5 \\ 2 & 2 \end{pmatrix}$                                       (D)  $\begin{pmatrix} -1 & -2 \\ -3 & 5 \end{pmatrix}$

19. Let  $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha \\ 1 & \alpha & \beta \end{pmatrix}$ . Which one of the following is equal to the  $\det(A)$ ?

- (A)  $(1-\alpha)(\beta-\alpha)$                                       (B)  $(1-\beta)(\alpha-\beta)$                                       (C)  $(1-\alpha)(\alpha-\beta)$                                       (D)  $(\beta-1)(\alpha-\beta)$

20. The solution set of the system  $\begin{cases} x-3y=5 \\ y+z=1 \\ -x+y-z=0 \end{cases}$  is

- (A)  $\{(2, -1, 2)\}$                                       (B)  $\{(5, 0, 1)\}$                                       (C)  $\{(-13, -6, 7)\}$                                       (D)  $\{ \}$

21. Let  $A$  and  $B$  be  $3 \times 3$  matrices such that  $A = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 5 & 0 \\ 0 & -1 & \frac{1}{2} \end{pmatrix}$  and  $|B| = \frac{1}{10}$ . Which one of the following is

equal to  $|2AB^T|$ ?

- (A) 1                                      (B) 4                                      (C) 100                                      (D) 400

22. Which one of the following is the simplest form of  $\frac{4-3i}{3+4i} + \overline{1-2i}$ ?

- (A)  $1+i$                                       (B)  $1+3i$                                       (C)  $2-i$                                       (D)  $1-3i$

23. In the set of complex numbers, the solution set of  $x^2 - 2x + 5 = 0$  is

- (A)  $\{2+4i, 2-4i\}$                                       (B)  $\{2+i, 2-i\}$                                       (C)  $\{1+2i, 1-2i\}$                                       (D)  $\phi$

24. For  $i = \sqrt{-1}$ , the simplest form of the expression  $i^{99} + i^{38} + 1$  is

- (A)  $-i$                                       (B)  $-i+1$                                       (C)  $-i+2$                                       (D)  $-2i$

25. The value of  $x$  and  $y$  that satisfy the equation  $(2x - yi)(3 + i) = 20i$
- (A)  $x = 0, y = -20$       (B)  $x = 3, y = 7$       (C)  $x = 1, y = -6$       (D)  $x = -5, y = 3$
26. The polar form of  $\frac{7-i}{3-4i}$  is
- (A)  $\sqrt{2} \left( \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)$       (C)  $2 \left( \cos \frac{\pi}{2} + i \sin \frac{\pi}{2} \right)$
- (B)  $\sqrt{2} \left( \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$       (D)  $2 \left( \cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$
27. A three-digit library ID-card is to be printed from the numbers 0,1,2,3,4,5 and 6 in a such a way that the first is non-zero and no number is to be repeated. How many such cards can be printed?
- (A) 180      (B) 216      (C) 210      (D) 343
28. A student needs to select three books from 4 different Mathematics, 4 different Physics and one Geography book. What is the probability that one of them is Mathematics and the other two are either Physics or Geography books?
- (A)  $\frac{10}{21}$       (B)  $\frac{19}{21}$       (C)  $\frac{13}{21}$       (D)  $\frac{17}{21}$
29. The following frequency distribution displays the age of students in a certain primary school.

AGE	8	10	11	12	13
NUMBER OF STUDENTS	5	15	8	10	2

Which one of the following is NOT true about the data?

- (A) The median is 10.5      (C) The mean is 10.6.  
 (B) The mode is 10.      (D) The range is 5.
30. If  $(\neg p \vee q) \Rightarrow (r \wedge \neg r)$  is true, then which of the following is necessarily true?
- (A)  $r \Rightarrow q$       (B)  $(q \Rightarrow p) \wedge r$       (C)  $p \wedge \neg r$       (D)  $(q \wedge r) \Rightarrow p$
31. If the truth value of  $(p \wedge \neg p) \Rightarrow [(q \vee \neg q) \Rightarrow r]$  is true, then which one of the following must be true?
- (A)  $p$       (B)  $\neg r$       (C)  $\neg q$       (D)  $q$
32. Suppose the proposition  $p \Rightarrow \neg q$  is false (F), which of the following is true?
- (A)  $\neg q \wedge (p \Rightarrow q)$       (B)  $\neg p \vee (q \Rightarrow \neg p)$       (C)  $(\neg q \vee p) \Leftrightarrow q$       (D)  $(q \vee p) \Leftrightarrow \neg p$
33. Which one of the following is NOT true about the graph of  $f(x) = \frac{x^2 - 1}{x^2 + 1}$  ?
- (A) The range of  $f$  is  $(-\infty, 1)$       (C) The line  $y = 1$  is a horizontal asymptote.  
 (B)  $f$  is an even function      (D) As  $x \rightarrow -\infty, f(x) \rightarrow 1$

34. If  $\vec{A} = (3, -3)$  and  $\vec{B} = (1, -3)$ , what is the unit vector in the direction of the unit vector in the direction of  $\vec{C} = 3\vec{A} - \vec{B}$ ?
- (A)  $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$       (B)  $\left(\frac{1}{\sqrt{10}}, -\frac{3}{\sqrt{10}}\right)$       (C)  $\left(\frac{4}{5}, -\frac{3}{5}\right)$       (D)  $\left(-\frac{6}{10}, \frac{8}{10}\right)$
35. If  $\vec{u}$  is a vector in the direction of the vector  $\vec{v} = (-3, 4)$  having length three times the length of the vector  $\vec{u} - \frac{3}{2}\vec{v} - 2\vec{j}$ , where  $\vec{j}$  is the unit vector in the direction of the positive y-axis?
- (A)  $\frac{1}{2}\sqrt{185}$       (B)  $\frac{1}{2}\sqrt{145}$       (C)  $\frac{21}{2}$       (D)  $\frac{17}{2}$
36. Consider the circle given by  $x^2 + y^2 = 2$  and the line  $\ell$  given by the parametric vector equation  $(x, y) = (2, 0) + t(-1, 1)$ . Which of the following is true?
- (A) The line  $\ell$  is tangent to the circle at  $\left(\frac{1}{2}, \frac{\sqrt{7}}{2}\right)$ .
- (B) The line  $\ell$  intersects with the circle at two distinct points.
- (C) The line  $\ell$  and the circle have no common points.
- (D) The distance from the center of the circle to the line  $\ell$  is  $\sqrt{2}$ .
37. Which one of the following is not equal to the expression  $(1 + \tan^2 x) \sin\left(x - \frac{3}{2}\pi\right)$ ?
- (A)  $\sec x$       (B)  $\cos x$       (C)  $-\sec x$       (D)  $-\cos x$
38. What are the period and amplitude of the function  $f(x) = \frac{1}{2} \sin\left(3 - \frac{2\pi}{3}x\right)$ , respectively?
- (A)  $-\frac{2}{3}\pi, \frac{1}{2}$       (B)  $\frac{2}{3}, 2$       (C)  $3, \frac{1}{2}$       (D)  $\frac{3}{2}\pi, 1$
39. The solution set of the equation  $\sin 3x = 1$  in the interval  $\left[0, \frac{\pi}{2}\right]$  is
- (A)  $\left\{\frac{\pi}{2}\right\}$       (B)  $\left\{\frac{\pi}{3}, \frac{\pi}{2}\right\}$       (C)  $\left\{\frac{\pi}{6}\right\}$       (D)  $\left\{-\frac{\pi}{2}, \frac{5\pi}{6}\right\}$
40. If  $\theta$  is a fourth quadrant angle and  $\sec \theta = \sqrt{2}$ , then what is  $\csc \theta$  equals to?
- (A)  $-\frac{\sqrt{2}}{2}$       (B)  $-\sqrt{2}$       (C)  $\frac{\sqrt{2}}{2}$       (D)  $\sqrt{2}$

**PART II: Fill in the blank spaces with a correct item that completes the sentence best.**

- The coefficient of the term  $x^2y^3$  in the expansion of  $(2x + 5y)^5$  is \_\_\_\_\_.
- The standard deviation of the dataset **20, 16, 12, 8, 18, 5, 9, 24** is \_\_\_\_\_.
- If  $\vec{u} = 3i + \frac{5}{2}j$  and  $\vec{v} = \frac{7}{2}i - \frac{1}{4}j$ , then the modulus of the vector  $w = 2\vec{u} - \vec{v}$  is equal to \_\_\_\_\_.
- The image of the circle  $x^2 + y^2 - 2x + 3y = 8$  after being reflected by the line  $y = 2x - 3$  is \_\_\_\_\_.
- If A is a square matrix of order 3 and  $\det(A) = 5$ , then the value of  $\det(A \times \text{adj}(A))$  is \_\_\_\_\_.

**PART III: Work out each of the following questions clearly and neatly. Answers without sufficient supporting work will receive no credit. (Total of 5 Points)**

1. Find the solution set of the trigonometric equation  $\sqrt{3}\sin 2x = \cos 2x$  in the interval  $[0, 2\pi]$ .
2. Find the values of A, B and C that make the following mathematical statement true for all  $x \neq 0$ .

$$\frac{x-1}{x^3+x} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

3. Given the second-degree equation  $3x^2 - 6x = y^2$ .
  - (a) Identify what the equation represents to.
  - (b) Sketch the graph of curve.

THE PLANET IS YOURS AND SO IS THE FUTURE, SO BE SAFE!



**GOOD LUCK**

