**WACHEMO UNIVERSITY COLLEGE OF NATURAL AND COMPUTATIONAL SCIENCE**

**Department of Physics**

# Physics Module for Pre-University Remedial Program for ESSLCE Examinees

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## Department of Physics

# Physics Module for Pre-University Remedial Program for ESSLCE Examinees


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# Contents

# Chapter 1

# Vectors

## 1.1 Vectors

Learning competencies

- Demonstrate an understanding of the difference between scalars and vectors and give common examples.
- Explain what a position vector is.
- Use vector notation and arrow representation of a vector.
- Specify the unit vector in the direction of a given vector.
- Determine the magnitude and direction of the resolution of two or more vectors using Pythagoras's theorem and trigonometry.
- Add vectors by graphical representation to determine a resultant.
- Add/subtract two or more vectors by the vector addition rule.
- Use the geometric definition of the scalar product to calculate the scalar product of two given vectors.
- Use the scalar product to determine projection of a vector onto another vector.
- Use the vector product to test for collinear and orthogonality vectors.
- Explain the use of knowledge of vectors in understanding natural phenomena.

### 1.1.1 Vector and Scalar quantities

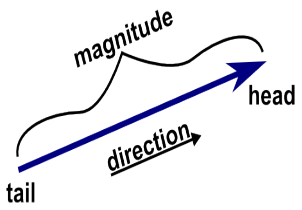
Scalars are the physical quantities that have the only magnitude. **Examples** of scalars are electric charge, density, mass etc. Vectors are physical Quatities that must be described by both magnitude and direction.
Example: **Velocity, Force, Torque, Electric field** etc.

### 1.1.2 Vector Representation

Vectors are represented in two methods (**Analytical/Algebric) and Graphical/Geometrical)**

1. **Analytical methods**: Vectors are representated analytically by a letter with an arrow over its head or with bold face letter.
   Example: Force $\Longrightarrow \overrightarrow{F}$ or **F**, Momentum $\Longrightarrow \overrightarrow{P}$ or **P**, Vector A $\Longrightarrow \overrightarrow{A}$ or **A**

2. **Graphical/Geometrical methods**: Graphically vectors are representated by a straight line and arrow drown to the scale. The length of the line is the magnitude

   of the vector and arrow tells us the direction.

### 1.1.3 Vector Addition and subsection

The sum of two or more vectos is called **resultant vector** ($\vec{R}$). Note that subtraction is addition of the negative ie

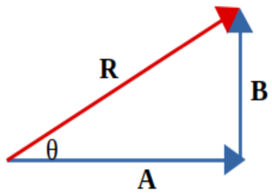
$$\vec{R} = \vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

Vector addition is not simple algebraic addition of numbers that is handled with the normal rules of arithmetic. It Obeys the laws of vector addition as follows

- The resultant of two vectos having the same direction is algebraic sum of the two vectors with the same direction as both.
  Example $\vec{A} = 8m$ East and $\vec{B} = 6m$ East then $\vec{R} = \vec{A} + \vec{B} = 14m$ East

- The resultant of two vectors having opposite direction has magnitude equal to the difference of magnitudes of the vectors and the resultant has the same direction as the larger vector. Example: $\vec{A} = 8m$ East and $\vec{B} = 6m$ West then $\vec{R} = \vec{A} + (-\vec{B}) = 2m$ East

- The resultant of two vectors acting at right angle with each other is obtained using Pythagorus theorem. Example: $\vec{A} = 8m$ East and $\vec{B} = 6m$ North then, And then the magnitude of $\vec{R}$ obtained using **Pythagorus theorem** as



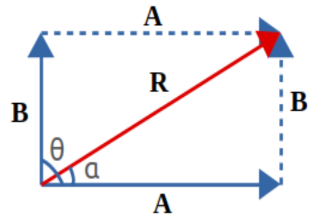

Triangular Methods:

Joining the vectors head to in a certain order, then the vector that is drown from the tail of the first vector to the head of the last vector is resultant vector (R)

OR

Parallelogram methods

Dr-owing the vectors from the same origin and then complete the parallelogram. The resultant vector R is the diagonal of the parallelogram.




$$R^2 = A^2 + B^2 + 2ABcos\theta = A^2 + B^2 + 2ABcos90^0 = A^2 + B^2$$

$R = \sqrt{(8m)^2 + (6m)^2} = 10m$

Direction of $\vec{R}$ obtained by **trigonometery**

$$tan\theta = \frac{Opp.}{adj.} = \frac{|B|}{|A|}$$

$$\theta = tan^{-1}(\frac{B}{A}) = tan^{-1}(\frac{6}{8}) = 36.87^o$$

- If the two vectors inclined at a certain anle $\theta$ to each other.

  The magnitude of the vector $\vec{R} = \vec{A} + \vec{B}$ is given by

$$|\vec{R}| = \sqrt{A^2 + B^2 + 2ABcos\theta}$$

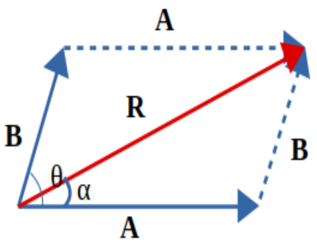
The magnitude of the vector $\vec{R} = \vec{A} - \vec{B}$ is given by

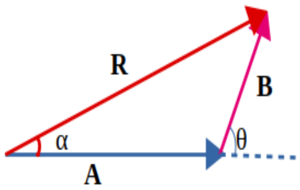
$$|\vec{R}| = \sqrt{A^2 + B^2 - 2ABcos\theta}$$

And its direction is given by $\alpha = tan^{-1}(\frac{Bsin\theta}{A+Bcos\theta})$

## Parallelogram methods

Dr-owing the vectors from the same origin and then complete the parallelogram. The resultant vector R is the diagonal of the parallelogram.




## Triangular Methods:

Joining the vectors head to in a certain order, then the vector that is drown from the tail of the first vector to the head of the last vector is resultant vector (R)

Or using sin law $\frac{sin(180^0-\theta)}{R} = \frac{sin\alpha}{B}$ or $sin\alpha = \frac{sin(180^0-\theta)}{R}B$

$$\alpha = sin^{-1}(B\frac{sin(180^0-\theta)}{R})$$

NB:

- If the vectors form a closed polygon when joined head to tail in a certain order, their resultant is **zero or null vector**
- Two or more vectors are equal if and only if they are
  - the same physical quantities
  - have the same magnitude and
  - have the direction

**Class work**

1.Given Vector $\vec{A}$ and $\vec{B}$. Find the resultant vector $\vec{R}$= $\vec{A}$+$\vec{B}$

a) If the $\vec{A}$ = 4bunit East and $\vec{B}$ = 3unit East

b) If the $\vec{A}$ = 4bunit East and $\vec{B}$ = 3unit West

c) If the $\vec{A}$ = 4bunit East and $\vec{B}$ = 3unit north

d) If the $\vec{A}$ = 4bunit East and $\vec{B}$ = 3unit at $60^o$ north of east

2. A car travels 20.0km due north and then 35.0km in direction $60^0$ west of north. Find the magnitude and direction of the car's resultant displacement. (**ans** $\vec{S} = 48.2km$ **at** $39.0^o$ **west of north**).

### 1.1.4 Vector Components

Components of vectors are projection of vectors along coordinate axis (x, y, z-axis). This meanse splitting vector into its Components. Consider the following figures. From the figures we can see that $A_x$ and $B_y$ forms two sides of right angle triangle with hypotenuse of length A. Using simple trigonometery (definition of sin and cosin) we see that

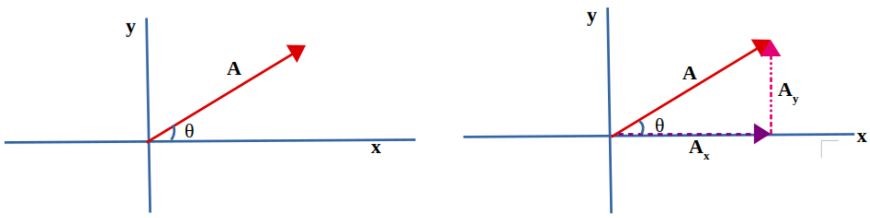



Figure 1.1: *Vector Components*

$$cos\theta = \frac{A_x}{|\vec{A}|} \Longrightarrow A_x = Acos\theta$$

$$sin\theta = \frac{A_y}{|\vec{A}|} \Longrightarrow A_y = Asin\theta$$

Thus $\vec{A} = \vec{A_x} + \vec{A_y}$ for three dimension $\vec{A} = \vec{A_x} + \vec{A_y} + \vec{A_z}$

### 1.1.5 Unit Vector

Unit vector is dimensionless vector with unit magnitude.

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$$

$\hat{A}$ read as A hat or caret is a unit vector that points in the direction of vector A. We shall use the symbols $\hat{i}$, $\hat{j}$ and $\hat{k}$ to representat a unit vector pointing in the positive x, y and z direction respectively as we can see from the figure above.

- The unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$ in rectangular coordinate System. $\hat{i}$, $\hat{j}$ and $\hat{k}$ are mutually perpendiculr axes.

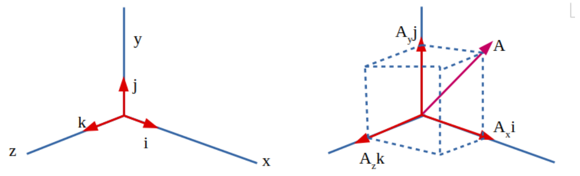



Figure 1.2: *Unit Vector in Rectangular Coordinat Axis*

- In general Vector A in rectangular coordinate system can be written as the sum of three vectors each which is parallel to a coordinate axes $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$

**Addition and Subtraction** of two vectors $\vec{A}$ and $\vec{B}$ can be written intermes of unit vector as

$$\vec{A} + \vec{B} = (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) + (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$$

$$= (A_x\hat{\imath} + B_x\hat{\imath}) + (A_y\hat{\jmath} + B_y\hat{\jmath}) + (A_z\hat{k} + B_z\hat{k}) = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} + (A_z + B_z)\hat{k}$$

**Class work**

1. Given vectors $\vec{A} = 4m\hat{\imath} + 3m\hat{\jmath}$, $\vec{B} = 2m\hat{\imath} - 3m\hat{\jmath}$, $\vec{C} = 2m\hat{\imath} + 3m\hat{\jmath} - 2m\hat{k}$ and $\vec{D} = 1m\hat{\imath} - 2m\hat{\jmath} + 2m\hat{k}$. Find

   a) $|\vec{A}|$ b) $2\vec{A} + \vec{B} - \vec{C}$

   c) Unit vector in the direction of vector R such that $2\vec{C} + \vec{B} - \vec{R} = 0$

2. A particle undergoes three consecutive displacements $\vec{d_1} = (15\hat{\imath} + 30\hat{\jmath} + 12\hat{k})cm$, $\vec{d_2} = (23\hat{\imath} - 14\hat{\jmath} - 5.0\hat{k})cm$, $\vec{d_3} = (-13\hat{\imath} + 15\hat{\jmath})cm$. Find

   a) The components of the resultant displacement and its magnitude

   b) Unit vector in the direction of resultant displacement

## 1.2 Multiplication of vectors

Vector multiplication refer to several operations between two (or more) vectors. It may concern any of the following articles:

- Scalar-vector multiplication
- Dot product
- Cross product

### 1.2.1 Scalar-vector multiplication

Multiplication of a vector by a scalar changes the magnitude of the vector, but leaves its direction unchanged. The scalar changes the size of the vector. The scalar "scales" the vector.
For example, If

$$\vec{A} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

Multiplied $\vec{A}$ by the scalar b is

$$b\vec{A} = b(a_x\hat{i} + a_y\hat{j} + a_z\hat{k}) = ba_x\hat{i} + ba_y\hat{j} + ba_z\hat{k}$$

**Scalar multiplication obeys the following rules**:

- Additivity in the scalar: $(c + d)\vec{v} = c\vec{v} + d\vec{v}$;
- Additivity in the vector: $c(\vec{v} + \vec{w}) = c\vec{v} + c\vec{w}$;
- Compatibility of product of scalars with scalar multiplication: $(cd)\vec{v} = c(d\vec{v})$;
- Multiplying by 1 does not change a vector: $1\vec{v} = \vec{v}$;
- Multiplying by 0 gives the zero vector: $0\vec{v} = 0$;
- Multiplying by -1 gives the additive inverse: $(-1)\vec{v} = -\vec{v}$.

### 1.2.2 Dot product

The dot product of two vectors is the magnitude of one times the projection of the second onto the first. The symbol used to represent this operation is a small dot at middle height $(\cdot)$, which is where the name "dot product" comes from. Since this product has magnitude only, it is also known as the scalar product. Mathematically defined as

$$\vec{A} \cdot \vec{B} = ABcos(\theta)$$

where $\theta$ the angle btween $\vec{A}$ and $\vec{B}$
Let

$$\vec{A} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

and

$$\vec{B} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$$

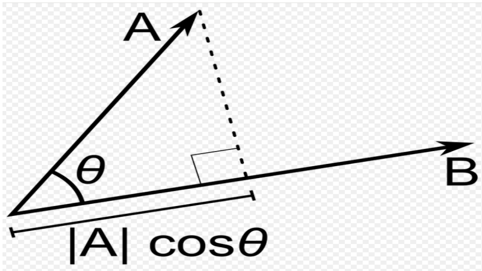



Figure 1.3: *Dot Product*

$$\vec{A}\cdot\vec{B} = (a_x\hat{i} + a_y\hat{j} + a_z\hat{k})\cdot(b_x\hat{i} + b_y\hat{j} + b_z\hat{k})$$

$$= a_xb_x\hat{i}\cdot\hat{i} + a_yb_y\hat{j}\cdot\hat{j} + a_zb_z\hat{k}\cdot\hat{k}$$

$$= a_xb_x + a_yb_y + a_zb_z$$

since $\hat{i}\cdot\hat{i} = (1)(1)cos(0) = 1$, $\hat{j}\cdot\hat{j} = (1)(1)cos(0) = 1$ , $\hat{k}\cdot\hat{k} = (1)(1)cos(0) = 1$ but $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{i} = (1)(1)cos(90^0) = 0$, $\hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{j} = (1)(1)cos(90^0) = 0$ , $\hat{k}\cdot\hat{i} = \hat{i}\cdot\hat{k} = (1)(1)cos(90^0) = 0$

**Dot Product Properties of Vector:**

- Dot product of two vectors is commutative i.e. $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$
- If $\vec{A}\cdot\vec{B} = 0$ then it can be clearly seen that either $\vec{A}$ or $\vec{B}$ is zero or $cos(\theta) = 0$.
- Also we know that using scalar product of vectors $(p\vec{A})\cdot(q\vec{B}) = (p\vec{B})\cdot(q\vec{A}) = pq(\vec{A}\cdot\vec{B})$
- The dot product of a vector to itself is the magnitude squared of the vector i.e. $\vec{A}\cdot\vec{A}) = AAcos(0) = A^2$
- Distributive Property: $\vec{A}\cdot(\vec{B} + \vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$
- Non-Associative Property: $\vec{A}\cdot(\vec{B}\cdot\vec{C}) \neq (\vec{A}\cdot\vec{B})\cdot(\vec{A}\cdot\vec{C})$, because the dot product between a scalar and a vector is not allowed.

### 1.2.3 Cross Product

The cross product of two vectors $\vec{a}$ and $\vec{b}$ is vector $\vec{c}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and equal magnitude to the area of the parallelogram between $\vec{a}$ and $\vec{b}$. The symbol used to represent this operation is a large diagonal cross ($\times$), which is where the name "cross product" comes from. Since this product has magnitude and direction, it is also known as the **vector product**.

$$\vec{a} \times \vec{b} = absin(\theta)\hat{n}$$

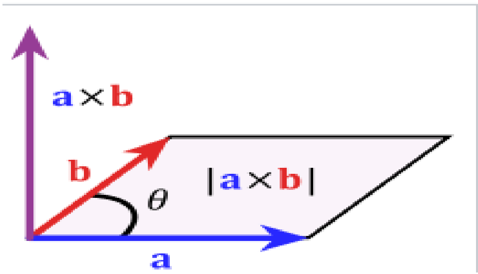


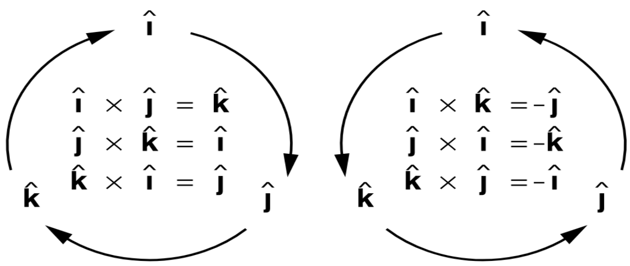

Figure 1.4: *Cross Product*

The vector $\hat{n}$ (n hat) is a unit vector perpendicular to the plane formed by the two vectors and $\theta$ is the angle between $\vec{a}$ and $\vec{b}$. The direction of $\hat{n}$ is determined by the right hand rule.

**Cross Product Properties** :

- the cross product is distributive: $\vec{a} \times (\vec{b} + \vec{c}) = (\vec{a} \times \vec{b}) + (\vec{a} \times \vec{c})$
- the cross product is not commutative: $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$

  but $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$
- the cross product of any vector with itself is zero: $\vec{a} \times \vec{a} = \vec{b} \times \vec{b} = 0$
- cross product of any unit vector with itself is zero: $\hat{i} \times \hat{i} = (1)(1)sin(0) = 0$, $\hat{j} \times j = (1)(1)cos(0) = 0$ , $\hat{k} \times \hat{k} = (1)(1)cos(0) = 0$
- any cyclic product of the three coordinate axes is **positive** and any anticyclic product is **negative** as shown bellow.




Let

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

and

$$\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$$

$$\vec{a} \times \vec{b} = (a_x\hat{i} + a_y\hat{j} + a_z\hat{k}) \times (b_x\hat{i} + b_y\hat{j} + b_z\hat{k})$$

$$\vec{a}\times\vec{b} = a_x\hat{i}\times b_x\hat{i}+a_x\hat{i}\times b_y\hat{j}+a_x\hat{i}\times b_z\hat{k}+a_y\hat{j}\times b_x\hat{i}+a_y\hat{j}\times b_y\hat{j}+a_y\hat{j}\times b_z\hat{k}+a_z\hat{k}\times b_x\hat{i}+a_z\hat{k}\times b_y\hat{j}+a_z\hat{k}\times b_z\hat{k}$$

$$\vec{a} \times \vec{b} = 0 + (a_xb_y)\hat{k} - (a_xb_z)\hat{j} - (a_yb_x)\hat{k} + 0 + (a_yb_z)\hat{i} + (a_zb_x)\hat{j} - (a_zb_y)\hat{i} + 0$$

$$\vec{a} \times \vec{b} = (a_yb_z - a_zb_y)\,\hat{i} + (a_zb_x - a_xb_z)\hat{j} + (a_xb_y - a_yb_x)\hat{k}$$

Or using determinat form
$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} \hat{i} + \begin{vmatrix} a_z & a_x \\ b_z & b_x \end{vmatrix} \hat{j} + \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix} \hat{k} = (a_yb_z - a_zb_y)\,\hat{i} + (a_zb_x - a_xb_z)\hat{j} + (a_xb_y - a_yb_x)\hat{k}$

<u>`Class work`</u>

1. Which of the following statements is true about the relation-ship between the dot product of two vectors and the product of the magnitudes of the vectors? (a) $\vec{A}\cdot\vec{B}$ is larger than AB; (b) $\vec{A}\cdot\vec{B}$ is smaller than AB; (c) $\vec{A}\cdot\vec{B}$ could be larger or smaller than AB, depending on the angle between the vectors; (d) $\vec{A}\cdot\vec{B}$ could be equal to AB.

2. Which of the following is equivalent to the following scalar product: $(\vec{A}\times\vec{B})\cdot(\vec{B}\times\vec{A})$? (a) $\vec{A}\cdot\vec{B} + \vec{B}\cdot\vec{A}$ (b) $(\vec{A}\times\vec{A})\cdot(\vec{B}\times\vec{B})$ (c) $(\vec{A}\times\vec{B})\cdot(\vec{A}\times\vec{B})$ (d) $-(\vec{A}\times\vec{B})\cdot(\vec{A}\times\vec{B})$

3. Which of the following statements is true about the relationship between the magnitude of the cross product of two vectors and the product of the magnitudes of the vectors? (a) $|\vec{A} \times \vec{B}|$ is larger than AB; (b) $|\vec{A} \times \vec{B}|$ is smaller than AB; (c) $|\vec{A} \times \vec{B}|$ could be larger or smaller than AB, depending on the angle between the vectors; (d) $|\vec{A} \times \vec{B}|$ could be equal to AB.

4. Is the triple product defined by $A \cdot (B \times C)$ a scalar or a vector quantity? Explain why the operation $A \cdot (B \times C)$ has no meaning.

5. Vector A is in the negative y direction, and vector B is in the negative x direction. What are the directions of (a) $A \times B$ $(b) B \times A$?

6. Given $\mathbf{M} = 6\hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $\mathbf{N} = 2\hat{\imath} - \hat{\jmath} - 3\hat{k}$ calculate the $\mathbf{M} \cdot \mathbf{N}$, $\mathbf{M} \times \mathbf{N}$ and the angle between $\mathbf{M}$&$\mathbf{N}$ and.

# Chapter 2

# Kinematics

Learning competencies

- Describe Kinematical terms such as distance, Displacement, average speed (velocities) and instantaneous speed (velocity).
- Solve numerical problems involving average velocity and instantaneous velocity.
- Derive equations of motion for uniformly accelerated motion.
- Apply equations of uniformly accelerated motion in solving problems.
- Relate scientific concepts to issues in everyday life.
- Explain the science of kinematics underlying familiar facts, observations and related phenomena.
- Describe the conditions at which freely falling bodies attain their terminal velocity.
- Define, Analyse and predict, terms in 2D motion
- Apply equations to solve problems related 2D motion.
- Distinguish between uniform and non-uniform circular motion.
- Analyse and predict, in quantitative terms, and explain uniform circular motion in the horizontal and vertical planes with reference to the forces involved.

## 2.1 Kinematics of the particle

The word Kinematics comes from Greek word "kinesis" meaning motion, thus

**Kinematics**: is a branch of mechanics that describes the motion of an object without reference to couse of motion (force). It does not give any information about force that couses it to move.

### 2.1.1 One or two dimensional (2D) motion

What is motion?
Motion is continuous change of position with time. Position is location of an object with respect to a choosen **reference frame or point**. Reference frame, also called frame of reference, in dynamics, system of graduated lines symbolically attached to a body that serve to describe the position of points relative to the body.
In physics we are considered three type of motion

1. **Translational motion**: is type motion in which all points (parts) of an object move the same distance in a given a given time. Example: A car moving in a straight line, a bullet which gets fired moves in rectilinear motion, child going down, a bird flying in the sky. In the above example, all the points of the body/object in motion are in the same direction. Translational motion can be of two types, rectilinear and curvilinear. Rectilinear motion is when an object in translational motion moves in a straight line motion. When an object in translational motion moves along a curved path, it is said to be in curvilinear motion

2 **Rotational motion**: is when an object moves about an axis and different parts of it move by different distances in a given interval of time. Examples: blades of a rotating fan, merry-go-round, blades of a windmill. When an object undergoes rotational motion, all its parts do not move the same distance in a given interval of time. For example, the outer portion of the blades of a windmill moves much more than the portion closer to the centre.

3 **Vibrational motion**: is when a body moves to and fro about its mean position is called vibratory motion. Vibratory motion can be described as any object moving/swinging back and forth, moving up and down, pulsating, etc. **Examples** Pendulums, swings, tuning forks, etc are of vibratory motion. Vibrational motion can be periodic or non-priodic

### 2.1.2 Motion in 1D

This is motion of a particle along straight line in fixed direction (or motion along one coordinate axis). Example: a car moving along a flat straight narrow road.


$\Delta x = x_f - x_i$

## Definition of Kinematical terms

<u>Distance and Displacement</u>

**Distance**: is the total path length covered by the moving object.

**Displacement**: is change of position i.e the shortest distance between start and end of motion. For example: a particle moving from point A to B as shown in figure below.

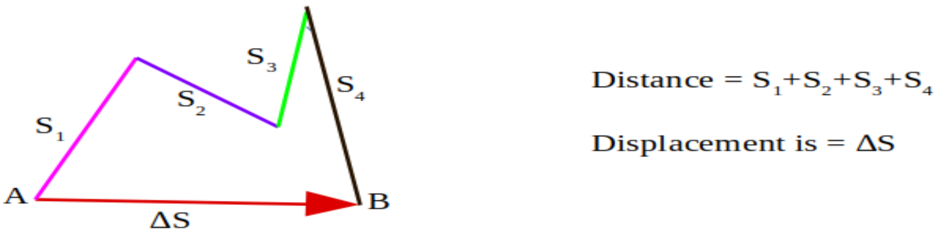



Figure 2.1: *Comparison of Distance and Displacement*

<u>Speed and Velocity</u>

**Speed (v)**: is the rate of change of distance in a unit time.

**Average Speed** ($v_{av}$): is total distance traveled by the total time required to cover the distance.

$$v_{av} = \frac{total\ \ distance}{total\ \ time}$$

**Velocity**($\vec{v}$): the rate of change of displacement as a function of time.

**Average Velocity** ($\vec{v}_{av}$): is change of displacement $\Delta x$ divided by the time interval $\Delta t$ during which the displacement occure.

$$\vec{v}_{av} = \frac{\vec{x}_f - \vec{x}_i}{t_f - t_i} = \frac{\Delta \vec{x}}{\Delta t}$$

<u>Instantaneous Velocity and Speed</u>

**Instantaneous Velocity v(t)**: is the Velocity of the particle at a given instant of time. It is the limit of average velocity as $\Delta t$ approaches to zero.

$$v(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \lim_{\Delta t \to 0} \frac{\vec{x}(t + \Delta t) - \vec{x}(t)}{\Delta t}$$

This can be rewritten as frist derivatives of displacement with respect time.

$$v(t) = \frac{dx}{dt}$$

The magnitude of Instantaneous velocity is instantaneous **speed**

`Class work`

1. Which of the following is true for displacement?
   (a) It cannot be zero.
   (b) Its magnitude is greater than the distance travelled by the object.
   (c) displacement may or may not be equal to distance
2. If the displacement of the body is zero, the distance covered by it may not be zero.
3. In which of the following cases of motions, the distance moved and the magnitude of displacement are equal ?
   (a) If the object is moving along straight road
   (b) If the object is moving along staight path
   (c) The pendulum is moving back and fro
   (d) The earth revolving around the sun
4. A particle moves along the x-axis according to the equation given below.

   $$\vec{x}(t) = (4 + 2t - t^2)m\hat{i}$$

   where t is in Second.
   a) Determine the displacement of this particle between the time interval t = 0 and t = 1s
   b) Determine the average velocity during those two time intervals
   c) Dirive a general expression for the instantaneous velocity as a function of time.
   d) calculate instantaneous velocity at t = 2s.
5. A boy walk from his home to school at constant speed of 5m/s along straight line and then back along the same line (road) from school to his home at constant speed of 6m/s.

a) What is his average speed?

b) What is his average velocity?

**Acceleration ($\vec{a}$):** is the rate of change of velocity.

**Average acceleration ($\vec{a}_{av}$):** is the change in velocity divided by time interval during which it occure.

$$\vec{a}_{av} = \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t}$$

**Instantaneous acceleration $\vec{a}(t)$:** is the acceleration of the particle at a given instant of time. It is the limit of average acceleration as $\Delta t$ approaches to zero.

$$\vec{a}(t) = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \lim_{\Delta t \to 0} \frac{\vec{v}(t + \Delta t) - \vec{v}(t)}{\Delta t}$$

This can be rewritten as Second derivatives of displacement or first derivatives of velocity with respect time.

$$\vec{a}(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

<u>`Class work`</u>

1. A particle moves along the x-axis component varies with time according to equation given below.

$$\vec{x}(t) = (20 - 2t + t^3)m\hat{i}$$

where t is in Second.

a) Determine initial position of the particle.

b) Determine the displacement of this particle between the time interval t = 0 and t = 1s; t = 1 and t = 4s

c) Determine the average velocity during those two time intervals

d) Dirive a general expression for the instantaneous velocity as a function of time.

e) calculate instantaneous velocity at t = 3s.

f) average acceleration between t = 2s to t = 3s.

g) Dirive a general expression for the instantaneous acceleration as a function of time.

h) calculate instantaneous acceleration at t = 2s.

### Uniform Motion in 1D

Uniform Motion: A body is said to be in a state of uniform motion if it travels equal distances in equal intervals of time. If the time distance graph is a straight line the motion is said to be uniform motion. This meanse that the velocity of the body remain constant as it cover equal distance in equal interval of time, in case of uniform rectilinear motion acceleration of the body will be zero. Here, the avrage speed and instantaneous speed will be equal to the actual speed; avrage velocity and instantaneous velocity will be equal to the actual velocity and the magintude of velocity is equal speed.

$$\vec{v}_{av} = \vec{v} = \vec{v}(t)$$

$$|\vec{v}| = v$$

$$\Delta x = \vec{v}(t)t$$

### Uniformely accelerated Motion in 1D

This is motion with constant acceleration ie velocity change with uniform rate.

$$\vec{a}(t) = \frac{dv}{dt} = \frac{v - v_i}{t} = constant$$

$$v(t) = v_i + at \tag{2.1}$$

Average velocity for uniformely accelerated motion is given by

$$\vec{v}_{av} = \frac{\vec{v} + \vec{v_i}}{2}$$

Thus

$$\vec{x} - \vec{x_i} = \Delta\vec{x}t = \vec{v}_{av}t = (\frac{\vec{v} + \vec{v_i}}{2})t \tag{2.2}$$

Using equation (2.1) into (2.2)

$$\Delta\vec{x} = (\frac{\vec{v_i} + at + \vec{v_i}}{2})t$$

$$\Delta\vec{x} = v_it + \frac{1}{2}at^2 \tag{2.3}$$

From equation (2.1)

$$t = \frac{v - v_i}{a} \tag{2.4}$$

Using equation (2.4) into (2.2)

$$\Delta x = (\frac{v + v_i}{2})(\frac{v - v_i}{a}) = \frac{v^2 - v_i^2}{2a}$$

$$v^2 = v_i^2 + 2a\Delta x \tag{2.5}$$

<u>`Class work`</u>

1. An electron in a cathod ray tube accelerate uniformely from $2.0 \times 10^4 m/s$ to $6.0 \times 10^6 m/s$ over 1.5cm.

   a) How long does the electron take the to travel this 1.5cm?

   b) What is its acceleration?

2. A track covers 40m in 8.5s while smoothely slowing down to a final speed of 2.8m/s. Find

   a) its original velocity

   b) its acceleration

3. A jet lands on an air craft at 140mi/hr and stops in 2s due to an arresting cable that snags the air plane.

   a) What is its acceleration?

   b) If the plane touches down at position $x_i = 0$ what is the final position of the plane?

4. A car traveling at constant speed of 45m/s passes a tropper hidden behind a billboard. One second after the speeding car passes the billboard, the tropper sets out from the billboard to catch it, accelerating at constant rate of 3.0m/$s^2$. How long does it take her to over take the car?

5. A jet plane lands with a speed of 100m/s and slow down at rate of 5m/$s^2$ as it comes to rest.

   a) What is the time interval needed by the jet to come to rest?

   b) Can this jet land on an airport where the runway is 0.8km long?

### Free falling bodies

A freely falling object is any object moving freely under the influence of gravity alone, regardless of its initial motion.

Example: object thrown upward or down ward and object released from rest.

Free fall is motion with constant gravitational acceleration $g = 9.81m/s^2$ toward the center of the earth. So we can use equation of uniformely accelerated as in table below

| **When released from rest** | **When thrown up** | **When thrown down** |
|---|---|---|
| $v_y = gt$ | $v_y = v_{oy} - gt$ | $v_y = v_{oy} + gt$ |
| $\Delta y = \frac{1}{2}gt^2$ | $\Delta y = v_{oy} - \frac{1}{2}gt^2$ | $\Delta y = v_{oy} + \frac{1}{2}gt^2$ |
| $v_y^2 = 2g\Delta y$ | $v_y^2 = v_{oy}^2 - 2g\Delta y$ | $v_y^2 = v_{oy}^2 + 2g\Delta y$ |

<u>`Class work`</u>

1. A girl thows a ball upwards, moving it an initial speed u = 15m/s. Neglect air resistance

   a) How long does the ball take to return to the girl's hand?

   b) What will be its velocity then?

2. A ball is thrown upward. While the ball is in free fall, does its acceleration (a) increase (b) decrease (c) increase and then decrease (d) decrease and then increase (e) remain constant?

3. After a ball is thrown upward and is in the air, its speed (a) increases (b) decreases (c) increases and then decreases (d) decreases and then increases (e) remains the same.

### 2.1.3 Two dimension (2D) motion

2D motion is motion in a plane (This meanse object moving along two coordinate axis simultaneously, and its position can be described by two coordinate). Example: Projectile motion, Circular motion

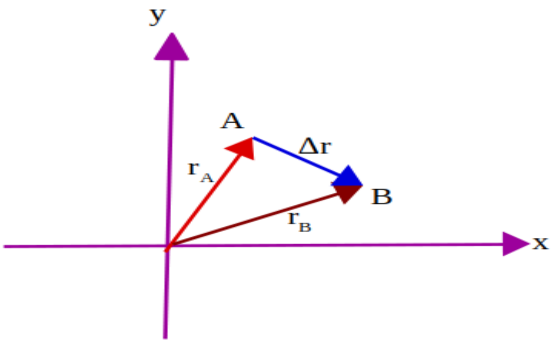



Figure 2.2: *Motion in a plane*

If a particle move from point A to point B in figure 2.2 its displacement is given by

$$\Delta\vec{r} = \vec{r}_B - \vec{r}_A$$

$$\Delta\vec{r} = (x_B\hat{i} + y_B\hat{j}) - (x_A\hat{i} + y_A\hat{j}) = \Delta x\hat{i} + \Delta y\hat{j}$$

For infintesmal change

$$d\vec{r} = dx\hat{i} + dy\hat{j}$$

Average velocity ($v_{av}$) is given by

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} = \vec{v}_x\hat{i} + \vec{v}_y\hat{j}$$

Instantaneous velocity is given by

$$v(t) = \lim_{\Delta t\to 0}\frac{\Delta r}{\Delta t} = \lim_{\Delta t\to 0}\frac{\vec{r}(t+\Delta t) - \vec{r}(t)}{\Delta t}$$

$$v(t) = \vec{v}_x(t)\hat{i} + \vec{v}_y(t)\hat{j}$$

Average acceleration is given by

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j}$$

Instantaneous acceleration

$$a(t) = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \lim_{\Delta t \to 0} \frac{\vec{v}(t + \Delta t) - \vec{v}(t)}{\Delta t}$$

$$a(t) = \vec{a}_x(t)\hat{i} + \vec{a}_y(t)\hat{j}$$

<u>Class work</u>

1. A bird flies in xy plane with velocity vector given by

$$\vec{v} = (\alpha - \beta t^2)\hat{i} + \gamma t\hat{j}$$

where $\alpha = 2.1 m/s$ and $\gamma = 2.8 m/s^2$ and the positive y direction is vertically upward at t = 0, the bird is at the origin.

   a) Determine average acceleration between time interval t = 0 to 1s

   b) calculate the general expression for instantaneous acceleration at any time t

   c) What is the birds altitude (y cordinate) as it flies over x = 0 for the first time after t = 0.

### 2.1.3.1 Projectile Motion

Projectile motion is motion of an object in a plan under the infuelence of gravity alone, regardless of its initial motion(neglacting air resistance). Examples: A ball kicked from the horizontal ground. The path followed by projectile motion is trajectory and downward

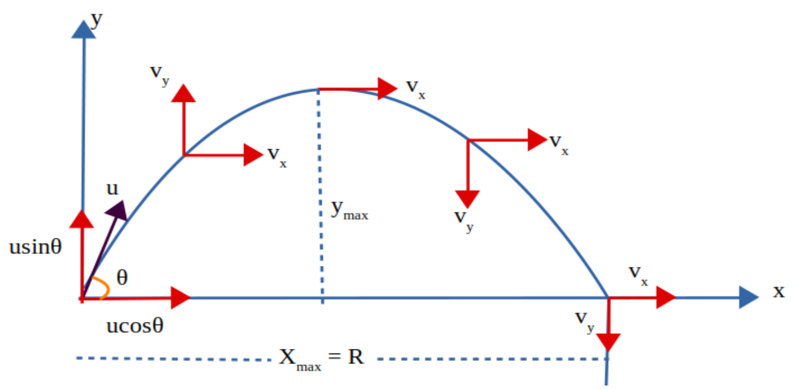



Figure 2.3: *Projectil Motion*

parabola due to gravitational acceleration and combination of horizontal and vertical velocity as we can see in figure 2.3. As projectile motion is 2D motion we can regard it as two separate and independent horizontal (x-component) and vertical (y-component) motion.

**Horizontal motion of projectile**

Horizontal motion of projectile motion is uniform motion (velocity constant, $a_x = 0$). Because no net force act on horizontal motion of projectile motion. ie

$$v_{0x} = v_0 cos\theta$$

this is horizontal component of initial velocity.

$$v_x = v_{0x} = v_0 cos\theta = constant$$

and

$$x(t) = v_{0x}t = v_x t = v_0 cos\theta t$$

**Vertical motion of projectile**

Vertical motion of projectile motion is uniformely accelerated motion. It is motion with constant gravitational acceleration of $g = 9.8m/s^2$ to ward the center of the earth. At the origin

$$v_{0y} = v_0 sin\theta$$

this is vertical component of initial velocity.
Now we can use equation uniformely accelerated motion as

$$v_y = v_{0y} - gt$$

$$\Delta y = \frac{(v_{0y} + v_y)}{2} t$$

$$\Delta y = v_{0y}t - \frac{1}{2}at^2$$

$$v_y^2 = v_{0y}^2 - 2g\Delta y$$

But at the maximum height $v_y = 0$ so

$$0 = v_{0y}^2 - 2gy_{max}$$

$$y_{max} = \frac{v_{0y}^2}{2g} = \frac{v_0^2 sin^2\theta}{2g}$$

Total time (t) is time of flight is given by

$$v_y = v_{0y} - gt$$

but at $y_{max}, v_y = 0$

$$0 = v_{0y} - gt_a$$

$$t_a = \frac{v_{0y}}{g} = \frac{v_0 sin\theta}{g}$$

But total time is $t = t_a + t_d$ and $t_a = t_d$ thus

$$t = \frac{2v_{0y}}{g} = \frac{2v_0 sin\theta}{g}$$

Range (R) is maximum horizontal displacement ($x_{max}$)

$$R = v_{ox}t = v_0 cos\theta(\frac{2v_0 sin\theta}{g})$$

$$R = \frac{v_0^2 2cos\theta sin\theta}{g}$$

$$R = \frac{v_0^2 sin2\theta}{g}$$

The maximum range is reached at an angle of projection $\theta = 45^0$

$$R = \frac{v_0^2 sin(2 \times 45^0)}{g} = \frac{v_0^2 sin(90^0)}{g} = \frac{v_0^2}{g}$$

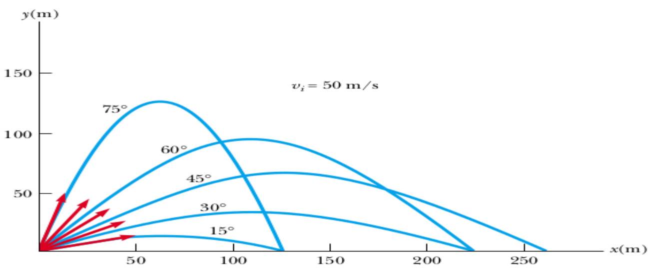



Figure 2.4: *A projectile launched from the origin with an initial speed of 50 m/s at various angles of projection. Note that complementary values of $\theta_i$ result in the same value of R (range of the projectile).*

<u>`Class work`</u>

1. A ball is kicked with an initial velocity if 40m/s from the ground at an angle of $30^0$ to the horizontal. (Use g = 10m/$s^2$) Calculate

   a) Horizontal and vertical component of initial velocity

   b) The vertical velocity after t = 1s, 2s, 3s, and 4s

c) Position ($\vec{r} = x\hat{i} + y\hat{j}$) after t = 1s, 2s, 3s, and 4s

d) Time of flight (total time)

e) maximum height

f) Range of projectile

2. An air plane moving horizontally with velocity of 500km/hr at a height of 2km above the ground dropped a bomb when it directly above the target. By how much distance will the bomb miss the target?

3. An astronaut on a strange planet finds that she can jump a maximum horizontal distance of 15.0m if her initial speed is 3.00m/s. What is the free-fall acceleration on the planet?

#### 2.1.3.2 Circular Motion

##### Uniform Circular Motion

**Uniform circular motion**: is a type of motion in which an object moves in a circular path at a constant speed. The direction of motion is constantly changing as the object moves around the circle. For example, imagine a car moving around a circular racetrack at a constant speed of 100 km/h. The car is always moving in a circle, and the direction of the car is constantly changing as it goes around the track. However, the car's speed is always the same, so the car is said to be in uniform circular motion. Another example of uniform circular motion is a planet orbiting around a star. The planet is constantly moving in a circular path around the star, and its speed is constant as it moves around the orbit.

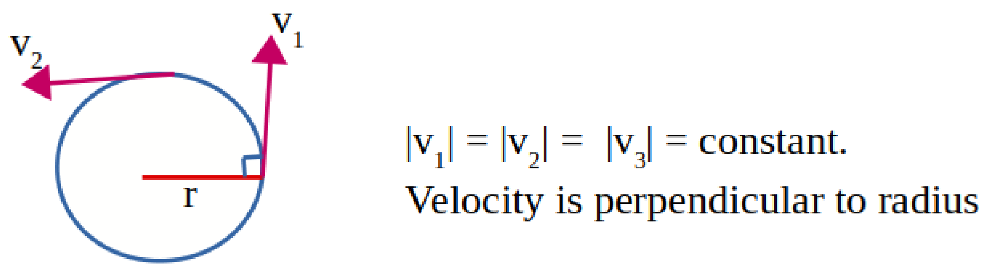
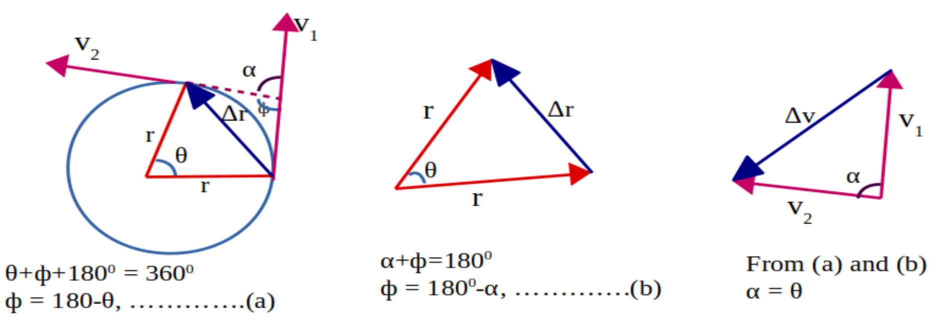



Figure 2.5: *Uniform Circular Motion.*

In uniform circular motion velocity is not constant because continuous variation of direction.




So the two triangle are similar by $S\hat{A}S$. For similar triangle the ratio of their side is equal

$$\frac{|\Delta\vec{r}|}{|\Delta\vec{v}|} = \frac{|\vec{r}|}{|\vec{v}|} \quad or \quad \frac{|\Delta\vec{v}|}{|\Delta\vec{r}|} = \frac{|\vec{v}|}{|\vec{r}|}$$

$$\Delta v = \frac{v}{r} \times \Delta r$$

Dividing bothside by $\Delta t$

$$\frac{\Delta v}{\Delta t} = \frac{v}{r} \times \frac{\Delta r}{\Delta t}$$

$$a_c = \frac{v}{r} \times v$$

$$a_c = \frac{v^2}{r}$$

Thus $a_c$ is **radial or centerpital** accelaration and it is always toward the center of the circle perpendiculr to velocity. Its magnitude is constant but its direction change continuous perpendiculr to velocity. This accelaration is due to a **centripetal force**. A centripetal force is a net force that acts on an object to keep it moving along a circular path and its direction is toward the center the circle. **Example:** The tension force in the string of a swinging tethered ball and the gravitational force keeping a satellite in orbit are both examples of centripetal forces. Multiple individual forces can even be involved as long as they add up (by vector addition) to give a net force towards the center of the circular path.
**Period (T)**: it is time taken for one complete rotation.

$$T = \frac{2\pi r}{v}$$

### Non-uniform Circular Motion

**Non-uniform circular motion** is a type of circular motion in which the speed of an object moving in a circular path changes at different points along the path. In other words, the magnitude of the velocity vector of the object is not constant, meaning that the object is accelerating even though it is moving in a circle.

An example of non-uniform circular motion is a car driving around a curved road. The car's speed may change as it navigates the curve, depending on factors such as the car's position on the curve and the road conditions. The car's direction is constantly changing as it moves around the curve, but the speed is not constant. As a result, the car is undergoing non-uniform circular motion. Another example of non-uniform circular motion is a planet orbiting a star, as the planet speeds up and slows down in its elliptical orbit. In this case there are two type of acceleration:-

1. **Radial or centerpital accelaration**:- due to change of direction of motion

$$\vec{a}_c = \frac{v^2}{r}$$

2. **Tangential accelaration**:- due to change magnitude of velocity. Its magintude is

change of velocity over change of time.

$$\vec{a}_T = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} = \frac{\Delta v}{\Delta t}$$

Its direction is in the direction of Velocity which is perpendicular to centerpital accelaration. Thus total accelaration

$$\vec{a} = \vec{a}_c + \vec{a}_T$$

Its magintude

$$\vec{a} = \sqrt{(\vec{a}_c)^2 + (\vec{a}_T)^2}$$

Its direction

$$\theta = tan^{-1}\left(\frac{a_T}{a_c}\right)$$

`Class work`

1. A ball tied to the end of a string 1m in length swings in a vertical circle under the infuelence of gravity. When the string makesan angle of $20^0$ its speed was 2m/s. Calculate

   a) magintude of centerpital accelaration

   b magintude of centerpital Tangential accelaration

   c) magintude and direction of of total accelaration

# Chapter 3

# Angular Motion

Learning competencies

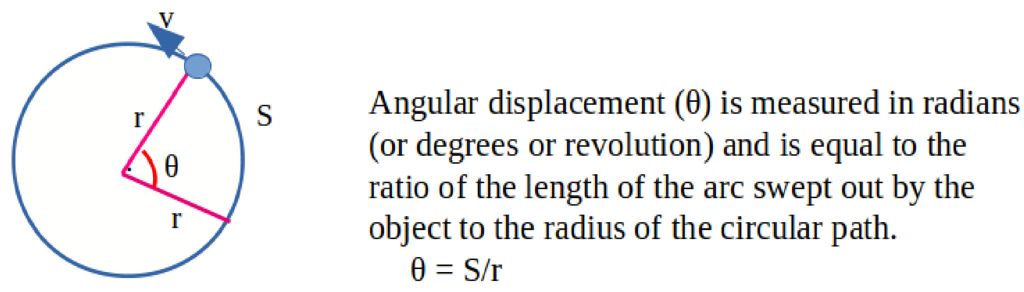
- Describe the rotational kinematical quantities.
- Give the angular speed and angular velocity of a rotating body.
- Determine the velocity of a point in a rotating body.
- Derive equations of motion with constant angular acceleration.
- Use equations of motion with constant angular acceleration to solve related problems.
- State the law of conservation of angular momentum.
- Apply the law of conservation of angular momentum in Understanding various natural phenomena, and solving problems.
- Express angular momentum as a cross product of r and p.
- Derive an expression for angular momentum in terms of I and $\omega$.
- Use the relationship between torque and angular momentum, according to Newton's second law.
- Apply the relationship between torque and angular momentum to solve problems involving rigid bodies.

## 3.1 Angular Motion

Angular motion is a type of motion that occurs when an object moves along a circular path or rotates around a fixed axis. Angular motion is characterized by two main quantities: angular displacement and angular velocity. Angular motion is important in many areas of physics, including mechanics, electromagnetism, and quantum mechanics. It is also used in many practical applications, such as in the design of engines, turbines, and other rotating machinery.

### 3.1.1 Rotational Kinematics

**Angular displacement**: is the change in the angle ($\theta$) of rotation of an object with respect to a fixed axis as we can see in figurebellow. Radian (rad) is SI unit of angular displacement,




one radian is angle sutended by an arc length equal to radius of the arc. The relation between revolutio(rev), degree(deg or $^0$) and radian (rad)

$$2\pi rad = 360^0 = 1rev \tag{3.1}$$

**Average Angular velocity**: is the rate of change of the angular displacement of an object with respect to time. Its represented by Greek letter $\omega$. It is measured in radians per second or (degrees per second or revolation per second) and is equal to the ratio of the change in the angular displacement of the object to the time interval over which the change occurred.

$$\vec{\omega} = \frac{\theta - \theta_0}{t - t_0} = \frac{\Delta\theta}{\Delta t}$$

**Instantaneous Angular Velocity**: It is average angular velocity as $\Delta t \twoheadrightarrow o$. This meanse angular velocity at instant of time (for infintesmal change)

$$\omega(t) = \frac{d\theta(t)}{dt}$$

**Angular acceleration**: the rate of change of the angular velocity of an object with respect to time. It is denoted by Greek letter $\alpha$.

**Average angular acceleration ($\omega_{av}$)**: is the ratio of andgular velocity to time interval $\Delta t$ during which the change occure.

$$\vec{\alpha} = \frac{\vec{\omega} - \vec{\omega}_0}{t - t_0} = \frac{\Delta\vec{\omega}}{\Delta t}$$

**Instantaneous Angular acceleration**: It is average angular acceleration as $\Delta t \twoheadrightarrow o$. This meanse angular acceleration at instant of time (for infintesmal change). Its SI unit is rad/$s^2$

$$\alpha(t) = \frac{d\omega(t)}{dt} = \frac{d^2\theta(t)}{dt^2}$$

### Uniformley Accelarated Rotational Motion

Uniformly accelerated angular motion refers to the motion of an object rotating around an axis at a constant rate of acceleration ($\alpha$). This means that the angular velocity of the object is changing at a constant rate over time. In this type of motion, the angular acceleration of the object is constant, which means that the rate of change of the angular velocity is also constant. The equation that describes this relationship is:

$$\omega = \omega_o + \alpha t \tag{3.2}$$

For uniformly accelerated angular motion the average angular velocity is given by

$$\omega_{av} = \frac{\omega + \omega_0}{2}$$

Therefore

$$\theta - \theta_0 = (\frac{\omega + \omega_0}{2})t \tag{3.3}$$

Using all those together

$$\theta = \omega_0 t + \frac{1}{2}\alpha^2 t^2 \tag{3.4}$$

and

$$\omega^2 = \omega_0^2 + 2\alpha\theta \tag{3.5}$$

<u>`Class work`</u>

1. The angular position of a fly wheel of car engines is given by

$$\theta = (2rad/s^3)t^3$$

   the diametre of the flywheel is 0.36m.

   a) Find the angle $\theta$ in radian, degree and revolution at time $t_1 2 secandt_2 = 5sec.$

   b) Find the distance that the particle of the rim moves during that time intervaly .

   c) Find the average angular velocity in rad/s, and rev./m

d) Find the general expression for the angular velocity at any time t.

e) Find the general expression for the angular accelaration at any time t.

2. A wheel rotates with angular acceleration of 3.5 rad/$s^2$. If the angular speed of the wheel is 2.0m/s at t=0.

   a) through what angle doese the wheel rotate in 2.0s

   b) what is the angular speed at t = 2.0s

### Angular Momentum

The angular momentum ($\vec{L}$) of a moving particle with respect to a given axis is given by

$$\vec{L} = \vec{r} \times \vec{p}$$

Where $\vec{r}$ is didtance from axis of rotation and $\vec{p} = m\vec{v}$ is linear momentum.

$$\vec{L} = \vec{r} \times \vec{p} = \vec{r} \times m\vec{v} = m\vec{r} \times \vec{v}$$

and we know that $v = r\omega$

$$\vec{L} = m\vec{r} \times r\vec{\omega} = mr^2\vec{\omega}$$

And we know that $I = mr^2$

$$\vec{L} = I\vec{\omega} \tag{3.6}$$

Eqn. 3.6 is angular momentum. Taking time derivatives of equation 3.6

$$\frac{d\vec{L}}{dt} = m\vec{r} \times \frac{d\vec{v}}{dt} = m\vec{r} \times \vec{a} = \vec{r} \times \vec{F}_{net} = \vec{\tau}_{net}$$

### Law of Conservation of Angular Momentum

This states that if the net external torque acting on the system is zero (the system is isolated ) then the angular momentum of the system is conserved (remain unchenged)

$$\vec{L}_i = \vec{L}_f$$

$$I_i\omega_i = I_f\omega_f \tag{3.7}$$

<u>`Class work`</u>

1. The position vector of a particle of mass 2kg is given as a function of time by $\vec{r} = (6\hat{i} + 5t\hat{j}$. Determine the angular momentum of the particle about the origin as a function of time.

2. A large circular disk of mass 2kg and radius 0.2m initially rotating at 50rad/s is coupled a smaller circular disk of mass 4kg and radius 0.1m initially rotating at 20rad/s in the same direction as large disk.

    a) Find the commen angular velocity after the disk are coupled.

    b) Calculate the loss of kinetic energy during this collision.

# Chapter 4

# Dynamics

Learning competencies

- Identify the four basic forces in nature.
- Define and describe the concepts and units related to force.
- Define the term dynamics.
- Define and describe the concepts and units related to coefficients of friction.
- Use the laws of dynamics in solving problems.
- Interpret Newton's laws and apply these to moving objects.
- Explain the conditions associated with the movement of objects at constant velocity.
- Solve dynamics problems involving friction.
- Analyse, in qualitative and quantitative terms, the various forces acting on an object in a variety of situations, and describe the resulting motion of the object.
- Describe the terms momentum and impulse.
- State the law of conservation of linear momentum.
- Discover the relationship between impulse and momentum, according to Newton's second law.
- Apply quantitatively the law of conservation of linear momentum.
- Distinguish between elastic and inelastic collisions.
- Describe head-on collisions.
- Describe glancing collisions.
- Define and describe the concepts and units related to torque.
- Describe centre of mass of a body.
- Determine the position of centre of mass of a body.
- Interpret Newton's laws and apply these to objects undergoing uniform circular motion.
- Solve dynamics problems involving friction.

## 4.1 Dynamics

**Dynamics**: In physics, dynamics is the branch of mechanics that deals with the study of motion and the forces that cause or affect that motion. It involves the analysis of how an object moves and the forces that cause it to move, including the study of the forces that cause changes in the motion of an object, such as acceleration, deceleration, and changes in direction. The fundamental concepts in dynamics are force, mass, and acceleration, as described by Newton's laws of motion. Dynamics is used to describe a wide range of physical phenomena, from the motion of particles at the subatomic level to the motion of planets in the solar system. It is used in many fields, including engineering, physics, and applied mathematics, to understand and predict the behavior of physical systems.

Force is a physical quantity that describes an interaction between two objects that can cause a change in motion of one or both objects. It is defined as the product of mass and acceleration, or more formally as the rate of change of momentum with respect to time. It is a push or a pull of an object (Intraction that change state of motion). We can't see force with our necked-eye but, in everyday life, we experience the following effects of force all the time.

- Force set or tends to set an object to motion
- Force stop or tends to stop motion
- Force change direction of motion
- Force accelerate or decelerate motion
- Force change shape and size of materials

### Type of force

Force usually catagorized into two

1. **Contact Force**: This is a force that requires physical contact between two objects in order for the force to be applied. **Examples**: Frictional force, Tension force, Normal force, Air resistance force, and Applied force.

2. **Non-contact Force**: This is a force that can act over a distance without any physical contact between the objects. **Examples**: Gravitational force, Magnetic force, Electrostatic force, Electromagnetic force, Nuclear force.

### Newton's Law of Motion

Newton's laws of motion are three fundamental principles that describe the behavior of objects in motion. They were first introduced by Sir Isaac Newton in his 1687 work "Philosophiæ Naturalis Principia Mathematica". The laws are:, 1. Newton's frist Law of motion (Law of inertia), 2. Newton's second law (Law of acceleration) and 3. Newton's third law (action and reaction force)

**Newton's first law**: This states that "Unless an external force exerted on the body the state of motion the body remain as it is". This is called law of inertia. **Inertia:** is the tedencey of the body to resist its change of state of motion.

**Newton's second law**: This states that "accelaration of an object is directely proportional to the net force acting on it and inversely proportional to is mass".

$$\sum \vec{F} = m\vec{a}$$

$$\sum \vec{F} = \sum \vec{F}_x + \sum \vec{F}_y + \sum \vec{F}_z = m(\vec{a}_x + \vec{a}_y + \vec{a}_z)$$

**Newton's third law**: This states that "if object A exert force on object B, then object B exert a force on object A that is equal in magnitude and opposite in direction". Thus force always occure in pair. This pair of force are called action and reaction force. For every action force there is reaction force. Action and reaction force are always:

- the same in magnitude
- opposite in direction
- act on different bodies
- the same type

<u>`Class work`</u>

1. Object of mass 10kg is exerted on by a force of $\vec{F_1} = (2\hat{\imath} + 3\hat{\jmath})N$, $\vec{F_2} = (4\hat{\imath} - 3\hat{\jmath})N$ and $\vec{F_3} = (-\hat{\imath} + 3\hat{\jmath})N$, calculate

   a) net force on this object

   b) its accelaration

2. A 2kg object undergoes an accelaration of given by $\vec{a} = (3\hat{\imath} + 4\hat{\jmath})m/s^2$. Find the magnitude resultant of force.

3. A particle of mass 2unit moves along space curve defined by $\vec{r}(t) = (4t^2 - t^3)\hat{\imath} - 5t\hat{\jmath} + (t^4 - 2)\hat{k}$. Find the force acting on it at any time t.

4. Find the force needed to accelarate a mass of 400kg from velocity $\vec{v}_0 = (4\hat{\imath} - 5\hat{\jmath} + 3\hat{k})m/s$ to $\vec{v}_f = (8\hat{\imath} + 3\hat{\jmath} - 5\hat{k})m/s$ in 10s.

### Friction Force

**Friction force**: is a force generated in opposite direction to the motion when solid object slide or attempt to slid over each other. Its magnitude is given by

$$f = F_N \mu$$

Where $\mu$ is coefficient of friction (constant that depend on the nature of the surface in contact), $F_N$ is normal force. There are two type of frictional force

- **Static friction**:- friction occure when object attempt to slid over each other but not yet slid over each other. Its magnitude given

  $$f_s = F_N \mu_s$$

  $\mu_s$ is coefficient of static friction

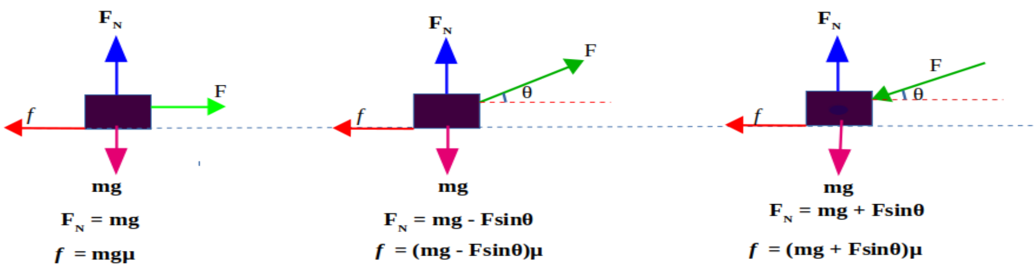
  **Kinetic friction**:- friction force occure when object sliding over eachother

  $$f_s = F_N \mu_k$$

  $\mu_k$ is coefficient of kinetic friction

NB $\mu_s > \mu_k$ thus $f_s > f_k$

**Normal Force ($F_N$)**:- Is a force or component of force that is perpendicular to the surface in contact and equal in magnitude to the force that holds the surface press together. Normal force equal to $mg$ when the sliding object is on horizontal surface and acted on by horizontal force as shown in figure below. But if the force acted on the object is at a certain angle to the horizontal the normal force is different as shown in figure below




<u>Class work</u>

1. A 20kg block is initially at rest on a horizontal surface. A horizontal force of 75N is a required to set the block in motion. After it is in motion a horizontal force of 60N is required to keep the block moving with constant speed. Find the coefficient of static and kinetic friction.

**Applied Force**:Applied force is a physical force that is applied to an object by a person or another object. It is a force that causes an object to move, accelerate, or change direction. Applied force is an important concept in physics and is used to describe many physical phenomena, including the motion of objects, the behavior of fluids, and the behavior of electromagnetic fields.

**Gravitational force** is the force by which a planet or other body draws objects toward its center. The force is always attractive and acts along the line connecting the two bodies. The force is proportional to the product of the two masses and inversely proportional to the square of the distance between them. The proportionality constant is known as the gravitational constant. The gravitational force is responsible for keeping the planets in orbit around the sun and for keeping the moon in orbit around the Earth. The mathematical formula for the gravitational force between two objects can be expressed using Newton's law of gravitation:

$$F = G\frac{m_1 m_2}{r^2}$$

where:

- F is the gravitational force between the two objects, measured in Newtons (N), - G is the gravitational constant, which has a value of approximately $6.674 \times 10^{-11} Nm^2/kg^2$, - $m_1$ and $m_2$ are the masses of the two objects in kilograms (kg), - r is the distance between the centers of mass of the two objects, measured in meters (m)

**A restoring force** is a force that acts to bring an object back to its original position after it has been displaced. In other words, it is a force that opposes displacement. For example, the force exerted by a spring when it is stretched or compressed is a restoring force. In a simple harmonic motion, restoring force is directly proportional to the displacement and acts in the direction opposite to the displacement.The mathematical expression of the restoring force for a spring is:

$$F = -kx$$

where F is the restoring force, x is the displacement from the equilibrium position, and k is the spring constant, which is a measure of the stiffness of the spring.
Another example of a restoring force is the force of gravity acting on a pendulum. The restoring force of a pendulum is given by:

$$F = -mgsin(\theta)$$

where F is the restoring force, m is the mass of the pendulum, g is the acceleration due to gravity, and $\theta$ is the angle between the pendulum and the vertical. In general, the mathematical expression of the restoring force depends on the specific physical system being considered, and can be derived from the laws of physics governing that system.

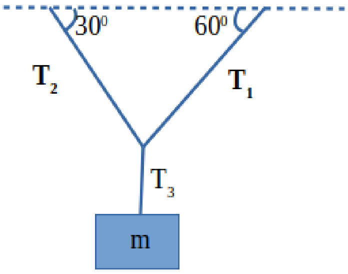
### Application and Newton's law of motion

In this case we apply Newton's law to objects either in equilibrium ($\vec{a} = 0$) or accelarating along straight line under action of constant force. The following procedure is recommanded when dealing problems with involving Newton's law.

1. Identify the object or particle on which force are exerted.
2. Identify the force exerted on the object (Draw free body diagram)
3. Decompose each force into their x,y and z-components.
4. Calculate net force, accelaration, velocity and so'on.

<u>Class work</u>

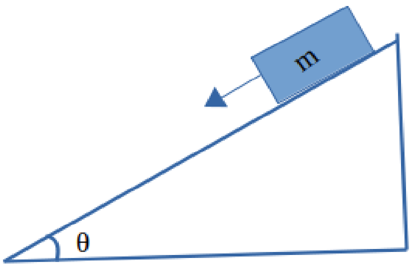
1. A block of mass 10kg hungs from three cords as shown below




What are the tension in the cords?
(Assume the cords has negligible mass)

Figure 4.1: *10kg hungs from three cords*

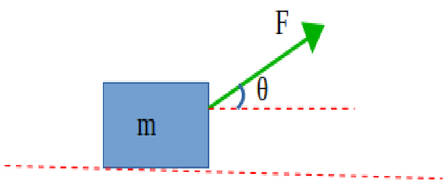
2. A block of mass m slids down an inclined plane as shown in the figure below.




Find the expression for the acceleration of the block if the inclined plane
a) is friction less
b) has coefficient of friction $\mu_k$

Figure 4.2: *mass m slids down an inclined plane*

3. The block of mass m sliding horizontally as shown in figure below.




Find the expression for the acceleration of the block, if the coefficient of friction between the block and the surface is $\mu_k$

Figure 4.3: *block of mass m sliding horizontally*

4. Two object of unequal mass are hung vertically over a frictionless pully of negligible mass as in figure below

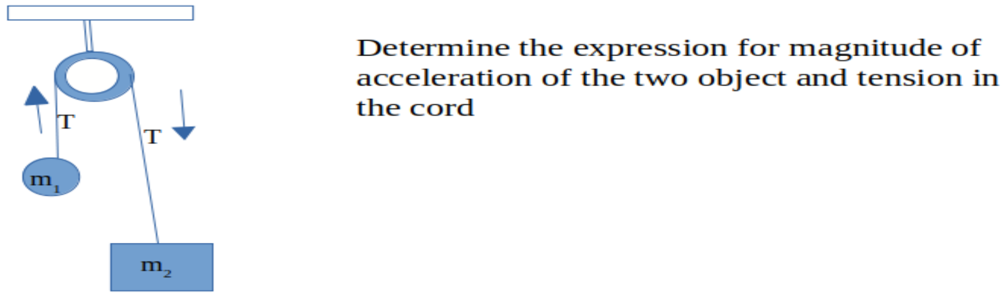



Figure 4.4: *unequal mass are hung vertically over a frictionless pully*

## 4.2 linear momentum

**Linear momentum** ($\vec{p}$) is defined as quality of an object to exert a force on any thing that tries to change its state of motion. Linear momentum is an important concept in physics because it is a measure of an object's ability to cause change through its motion. For example, a moving car has a lot of linear momentum and is able to do a lot of damage in a collision because it is difficult to stop. On the other hand, a stationary car has no linear momentum and is not able to cause much change through its motion. Its magnitude is the product of mass of the system with its velocity.

$$\vec{p} = m\vec{v} \tag{4.1}$$

For an objet's in three dimension

$$\vec{p} = m(v_x\hat{i} + v_y\hat{j} + v_z\hat{k}) \tag{4.2}$$

Linear momentum is a vector quantity, meaning it has both magnitude and direction. The direction of an object's linear momentum is the same as the direction of its velocity. Its SI unit is kgm/s.

**Impulse** ($\vec{J}$): Impulse is defined as the product of the force acting on an object and the time for which the force acts. Mathematically, impulse can be expressed as: `From Newton's second law`

$$\vec{F}_{net} = m\frac{\Delta v}{\Delta t} = \frac{\Delta}{\Delta t}(m\vec{v}) = \frac{\Delta \vec{p}}{\Delta t} \tag{4.3}$$

`This can be rewritten as`

$$\Delta\vec{p} = \vec{F}_{net}\Delta t = \vec{J} \tag{4.4}$$

Or

$$\vec{F}_{net} = \frac{\Delta \vec{p}}{\Delta t} \tag{4.5}$$

This is the relation between $\vec{p}$ and resultant force acting on it.

### 4.2.1 Conservation of Momentum

When ever two or more particles in an isolated system (in which net external force acting on the system is zero) intract, the total momentum of the system remain constant (conserved) i e

$$\sum \vec{p_i} = \sum \vec{p_f} \tag{4.6}$$

$$m_1\vec{u_1} + m_2\vec{u_2} = m_1\vec{v_1} + m_2\vec{v_2} \tag{4.7}$$

Where $u_1$ & $u_2$ are initial velocity of $m_1$ and $m_2$ respectively, and $v_1$ & $v_2$ are final velocity of $m_1$ and $m_2$ respectively Thus

$$\vec{p}_{xi} = \vec{p}_{xf}, \vec{p}_{yi} = \vec{p}_{yf}, \vec{p}_{zi} = \vec{p}_{zf}$$

<u>`Class work`</u>

1. A 60 kg archer stands at rest on a frictionless ice and fires a 0.5 kg arrow horizontally at 50m/s. With what velocity doese archer move across the ice after firing the arrow?

### 4.2.2 Collision

**Collision**: is the event of two particles comming together for short time and thereby producing impulsive force on each other. Collisions are an important topic in physics because they can be used to understand a wide range of phenomena, from the behavior of subatomic particles to the motion of celestial bodies in the universe. Depending on kinetic energy, the Q-value and coefficient of restitution Collision grouped in two (1) Elastic Collision, and (2) Inelastic Collision.

#### Elastic Collision

It is type of collision in which both kinetic energy and momentum are conserved.

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \tag{4.8}$$

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \tag{4.9}$$

The collision in which kinetic energy is fully conserved is called **perfectely elastic collision**.

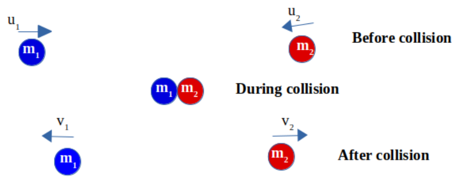



Figure 4.5: *perfectely elastic collision*

### Inelastic Collision

It is a type of collision in which only momentum is conserved but kinetic energy is not conserved.

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \tag{4.10}$$

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 \neq \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \tag{4.11}$$

A collision in which a colliding object stick together after collision is called **perfectely inelastic collision** In this collision kinetic energy lost as a form of heat and sound during

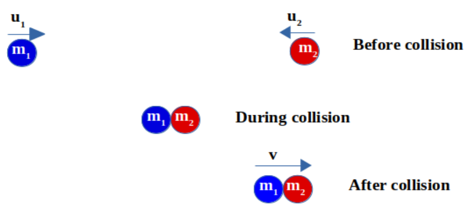



Figure 4.6: *perfectely inelastic collision*

collision. This lost of kinetic enegy represented by Q-value. The Q-value is equal zero for elastic collision and less than zero ($Q < 0$) for inelastic collision. Coefficient of restitution (e) $= \frac{u_2-u_1}{v_2-v_1}$ is equal 1 for elastic collision and zero for inelastic collision.

**Head-on Collisions:** Collision, when objects rebound on straight line paths that co-incide with original direction of motion. These collisions can be treated one dimensionally.

**Glancing Collisions**: When Object do not collide on the same path line they make glancing collision. To solve this problem, break it into components as shown in figure bellow.

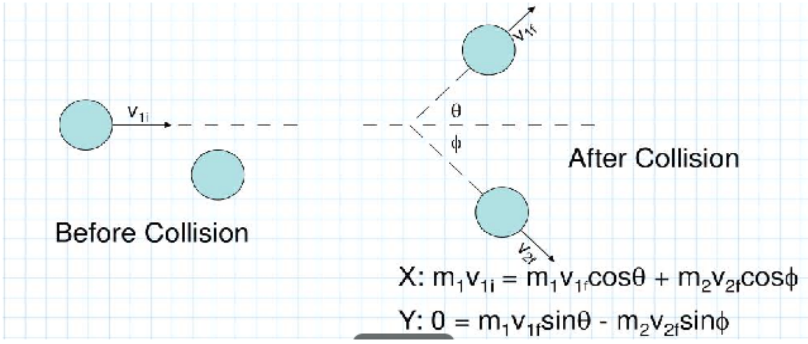



Figure 4.7: *Glancing Collision*

Class work

1. A ball of mass 2 kg is moving with a velocity of 12m/s collides with a stationary ball of mass 6 kg and comes to rest. Calculate velocity of ball of mass 6kg after collision.

2. A 10.0g bullet is fired into a stationary block of wood (m = 5.00kg). The bullet sticks into the block, and the speed of the bullet-plus combination immedately after collision is 0.600m/s. What was the original speed of the bullet?

3. A block of mass $m_1 = 1.6kg$ initially moving to the right with a speed of 4m/s on a horizontal frictionless track collides with a block of mass $m_2 = 2.1kg$ initially moving to the left with speed of 2.5m/s. If the collision is elastic, find the velocities of the two block after collision?

4. A partcle of mass 4.0kg initially moving with velocity of 2.0m/s collides with a partcle of mass 6.0kg, initially moving velocity of -4m/s. What are the velocity of the two particle after collision?

5. A 4kg block moving right at 6m/s collides elastically with a 2kg moving at 3m/s left, find final velocities the blocks.

## 4.3 Center of Mass and Moment of Inertia

### 4.3.1 Center of Mass

The center of mass of an object or system is the unique point at which the entire mass of the object or system can be considered to be concentrated.It is the point about which the object or system will balance if it is supported at that point, and it is the point around which the object or system will rotate if it is free to do so. In a system of particles, the center of mass is the average position of all the particles in the system, weighted according to their masses. It is a useful concept in mechanics because it allows the analysis of the motion of an object or system as if all of its mass were concentrated at a single point. It is located somwher on the line joining the partcle and closser to the larger mass. The center of mass of an object or system can be found by taking the sum of the positions of all the particles in the system multiplied by their masses, and then dividing by the total mass of the system. Center of mass of several partcle with mass $m_1, m_2, \cdots, m_n$ at a distance $\vec{r}_1$, $\vec{r}_2$, $\cdots$, $\vec{r}_n$ from each other is given by

$$\vec{r}_{cm} = \frac{\sum_{i=1}^{n} m_i \vec{r}_i}{\sum_{i=1}^{n} m_i} = \frac{\sum_{i=1}^{n} m_i \vec{r}_i}{M} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots + m_n\vec{r}_n}{m_1 + m_2 + \cdots + m_n} \tag{4.12}$$

Where

$$M = \sum_{i=1}^{n} m_i = \text{total mass}$$

For coordinate x, y and z, center of mass given by

$$\vec{x}_{cm} = \frac{\sum_{i=1}^{n} m_i x_i}{M}\hat{\imath}, \quad \vec{y}_{cm} = \frac{\sum_{i=1}^{n} m_i y_i}{M}\hat{\jmath} \; and \; \vec{z}_{cm} = \frac{\sum_{i=1}^{n} m_i z_i}{M}\hat{k} \tag{4.13}$$

This is known as the center of mass formula.The concept of center of mass is closely related to other important concepts in mechanics, such as the center of gravity, which is the point at which the gravitational force acting on an object or system can be considered to be concentrated. In many cases, the center of mass and the center of gravity of an object or system are at the same location, but this is not always the case, especially for objects or systems that are not symmetrical.

### 4.3.2 Moment of Inertia

Moment of Inertia: is the a measure of body's resistance to rotational motion about about a particular axis. It is typically denoted by the symbol I and is measured in $kg * m^2$. Its magnitude is affected by distribution of mass of the body in relation to its axis of rotation. Thus there is no single value of moment of inertia of an object. But for a point mass moment of inertia is given by

$$I = mr^2$$

Moment of inertia of a rigid object, made up of a particles of mass $m_1, m_1, ...$ at respective distance $r_1, r_2, ...$ from its axis of rotation, its moment of inertial about that axis is given by

$$I = m_1 r_1^2 + m_2 r_2 + ...m_i r_i^2 = \sum_i m_i r_i^2$$

For continuous mass distribution

$$I = \int_{m_0}^{m} r^2 dm$$

In general moment of inertia of a body depend on

- size of the body
- Shape of the body

For example consider disk and sphere of the same mass and the same radius.
$I = \frac{1}{2}mr^2$ for uniform disk
$I = \frac{2}{5}mr^2$ for uniform sphere

- It also depend on a point of axis of rotation

Example: For uniform rod axis of rotation through its center

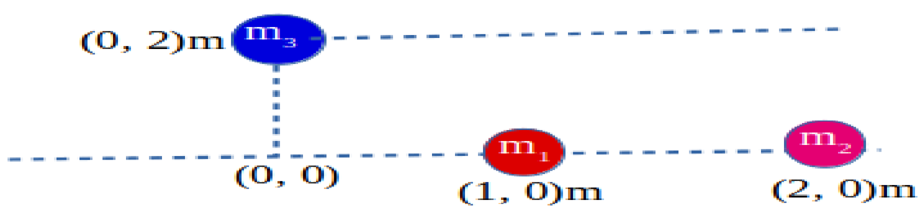
$$I = \frac{1}{12}ML^2$$

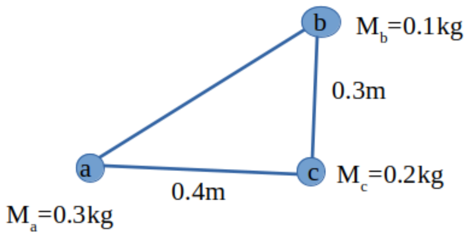
For uniform rod axis of rotation through its one end

$$I = \frac{2}{3}ML^2$$

<u>`Class work`</u>

1. A system consists three partcles of $m_1 = 1kg$, $m_2 = 1kg$ and $m_3 = 2kg$ located as in figure below. Find the center of mass of the system.






Find moment of inertia about an axis

a) through point a perpendicular to the plane formed by the particle.
b) along with rod bc

2. A partcles are connected by a light rod as shown bellow

3. Calculate moment of inertia of 4 equal masses be each having a mass of 50g and situated at the corner of the square of side 30cm. when axis of rotation passes through

   a) the center of the square perpendicular to the plane of the square

   b) the center of the square perpendicular to sides of the square.

   c) along the side of the square.

## 4.4 Torque and angular momentum

Torque: is defined as rotational effect of force ie (it is measure of force that couse an object to rotate around an axis. Torque is represented by symbol (greek letter) $'\tau'$. Torque is vector quality with both magintude and direction. It is calculated by the product of force and perpendicular distance from its axis of rotation.

$$\vec{\tau} = Frsin(\theta)\hat{n} = \vec{F} \times \vec{r}$$

Where $\theta$ is the direction of line of action of force $\hat{n}$ is unit vector in the direction of torque (clockwise or anticlockwise), r is moment arm of force (point of application of force from axis of rotation) F is applied.

Magintude of torque is depend on

- Size of force
- moment arm of force (radius or point of application of force)
- line of action of force (direction of line of action of force or $\theta$)

**Torque has maximum value when** $\theta = 90^0$ **and zero when** $\theta = 0^0 or 180^0$

**Torque intermes of angular accelarate** From definition of Torue we have

$$\tau = Fr \perp$$

From Newton's $2^{nd}$ law $F = a_T m$ for a particle moving in circle of radius but $a_T = \alpha r$ where $\alpha$ is angular accelaration Using those all together

$$\tau = \alpha rmr = \alpha(mr^2)$$

The term in the bracket is moment of inertia (I) so

$$\tau = I\alpha$$

<u>`Class work`</u>

1. A force of $\vec{F} = (2\hat{\imath} + 3\hat{\jmath}$ is applied to an object that is pivoted about a fixed axis that is aligned along the z-axis. If the force is applied at a point located at $\vec{r} = (3\hat{\imath} - 2\hat{\jmath})$

## 4.5 Conditions of Equilibrium (First and second)

Condition of Equilibrium in physics refers to state to the state where an object or system is not accelarating that is, its velocity is constant or zero (static equilibrium:-system is stable and at rest, the net torque must also be zero ). There are two main conditions of equilibrium.

1 **First condition of equilibrium**: This states that, the net force acting object must be zero. This meanse that the vector sum of all force acting on the object must be equal to zero. Mathematically this can be expressed as

$$F_{net} = \sum F = \sum F_x + \sum F_y + \sum F_z = 0$$

2 **Second condition of equilibrium**: This states that, the net torque acting on the object must be zero. This meanse that the vector sum of all the torque acting on the object must be equal to zero. Mathematically this can be expressed as

$$\tau_{net} = \sum \tau = \sum \tau_{clockwise} + \sum \tau_{anticlockwise} = 0$$

**Practical Example**

Two kids balancing a seesaw satisfy both conditions for equilibrium. In the figure 4.8, we see the lighter child sitting farther away from the pivot to create a torque equal in magnitude to that of the heavier child.

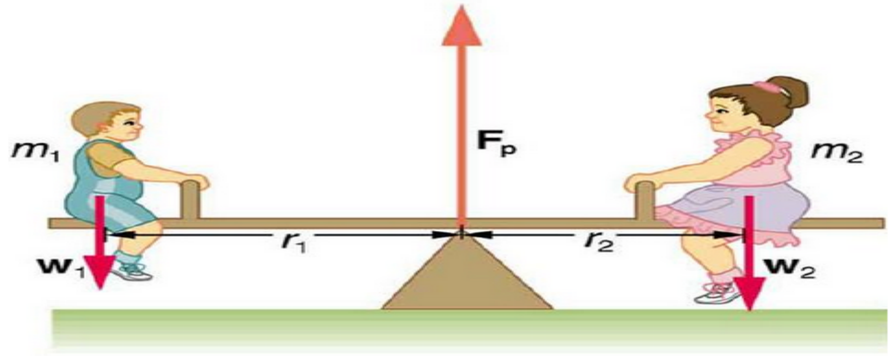



Figure 4.8: *seesaw satisfy both conditions for equilibrium*

# Chapter 5

# Work, Energy and Power

- Define work, kinetic energy and potential energy
- Calculate the work done by a constant force
- Derive work-kinetic energy theorem and apply in solving related problems
- State the principle of conservation of mechanical energy
- Solve problems related to the topics discussed in this section

## 5.1 Work done by constant and variable forces

Activity 5.1

What is work done?

Work done is defined as the magnitude of the force exerted in the direction of the displacement (or distance moved) multiplied by the displacement. Therefore for work to be done on an object, three essential conditions should be satisfied:

- Force must be exerted on the object
- The force must cause a motion or displacement
- The force should have a component along the line of displacement

Both force and displacement moved are vectors. Work done is the scalar product of force and displacement:

$$W = \vec{F} \cdot \vec{d} = Fdcos\theta$$

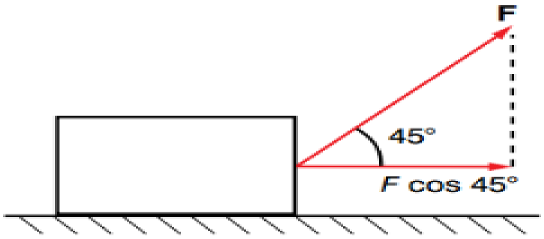



Figure 5.1: *Forces on a free body diagram*

where F and d are the magnitudes of the vectors. Work done is a scalar - it does not have directional properties. We can show the forces on a free body diagram (Figure 5.1). The force in the direction of the displacement is $Fcos45^0$, so work done is $W = Fdcos\theta$ which is the equation for the scalar product of the force and displacement vectors. The work done by a force on a moving object is zero when the force applied is perpendicular to the displacement of its point of application. That is, if $\theta = 90^0$, then W = 0 because $cos90^0 = 0$. The sign of the work also depends on the direction of F relative to d. The work done by the applied force is positive when the projection of F onto d is in the same direction as the displacement. When the projection of F onto d is in the direction opposite the displacement, W is negative. Remember that the unit of energy is the joule.

$$1joule = 1Newton \times 1meter$$

So 1 joule is the work done when a force of 1 Newton moves through a distance of 1 meter. This is the definition of the joule.

`Example`:

A man cleaning a floor pulls a vacuum cleaner with a force of magnitude F = 50.0 N at an angle of $30.0^0$ with the horizontal (Fig. 5.2). Calculate the work done by the force on the vacuum cleaner as the vacuum cleaner is displaced 3.0m to the right


Figure 5.2: *A vacuum cleaner*

`Solution`

A free body diagram as shown in Figure 5.2b. Using the definition of work.

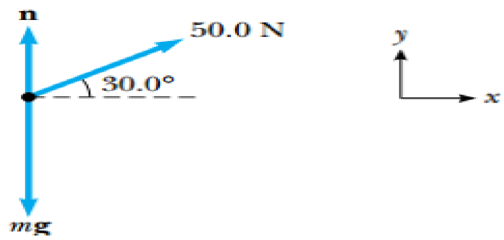



Figure 5.3: *Free-body diagram*

$F = 50.0N$ , $d = 3.0m$ $\theta = 30.0^0$ $cos30.0^0 = 0.866$

$$W = Fdcos\theta = (50.0N)(3.00m)(cos30.0^0) = (50.0N)(3.00m)(0.866) = 130J$$

To finalize this problem, notice in this situation that the normal force n and the gravitational $F_g = mg$ do no work on the vacuum cleaner because these forces are perpendicular to its displacement.

`Activity 5.2`

What are the differences between work done by constant and variable forces?

### 5.1.1 Work done by a variable force

A force is said to perform work on a system if there is displacement in the system upon application of the force in the direction of the force. In the case of a variable force, integration is necessary to calculate the work done. The work done by a constant force of magnitude F, as we know, that displaces an object by $\Delta x$ can be given as :

$$W = F \cdot \Delta x$$

In the case of a variable force, work is calculated with the help of integration. For example, in the case of a spring, the force acting upon any object attached to a horizontal spring can be given as:

$$F_s = -kx$$

Where, k is the spring constant, x is the displacement of the object attached. We can see that this force is proportional to the displacement of the object from the equilibrium position, hence the force acting at each instant during the compression and extension of the spring will

be different. Thus, the infinitesimally small contributions of work done during each instant are to be counted in order to calculate the total work done.
Therefore,

$$W_s = \int_{x_0}^{x} F_s(x) \cdot dx$$

$$W_s = -\int_{x_0}^{x} kxdx) = -k\left[\frac{1}{2}x^2\right]_{x_0}^{x}$$

$$W_s = -(\frac{1}{2}kx^2 - \frac{1}{2}kx_0^2) = -\frac{1}{2}k\Delta x^2$$

When a force varies, we cannot use the equation work done = force × distance moved. But the relationship for the area under the graph is still true. If we are able to record the force used and the displacement and plot a graph, we could find the work done by finding the area under the graph, as shown in Figure 5.3 You can estimate the average force by putting

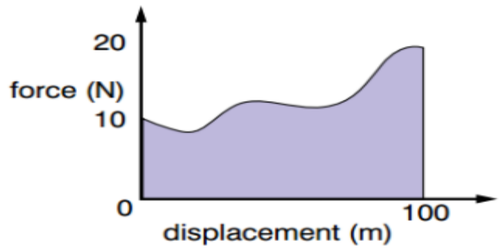



Figure 5.4: *Graph of variable force against displacement.*

a ruler on top of the graph as though you were going to draw a horizontal line. Adjust the position of the ruler so that the area between the graph line and the ruler is about the same above the ruler as it is below the ruler – this will give you an estimate of the average force

`Example`

A force F = 2x + 5 acts on a particle along the displacement. Find the work done by the force during the displacement of the particle from $x_0 = 0m$ to $x = 2m$. Given that the force is in Newton's.

`Solution`

Work done

$$W = \int_{x_0}^{x} F(x) \cdot dx = \int_{x_0}^{x} F(x)dxcos(0) = \int_{x_0}^{x} F(x)dx$$

$$W = \int_{0}^{2} (2x+5)dx = \left[\frac{2x^2}{2} + 5x\right]_0^2 = (2 \times 2 + 5 \times 2)Nm = 14J$$

## 5.2 Conservation of energy

In physics the term work (or often work done) is another way of saying energy is being transferred from one object to another or transformed from one type to another.

Work done = energy transferred

The more energy transferred the more work has been done. We define the sum of kinetic and potential energies as mechanical energy:

$$E = U + KE$$

Where E is the total mechanical energy, KE is the kinetic energy and U is the all types of potential energy. So we can write the general form of the definition for mechanical energy without a subscript on U In a system the mechanical energy of the system stays constant unless there is a force such as friction acting on the system.

The potential energy can be gravitational potential energy or energy stored in a spring, for example. When a spring is stretched, work is done because a force has been used to move one end of the spring by a certain displacement. Work is also done against gravity when you walk up stairs and you gain gravitational potential energy. When you walk down stairs, work is done by gravity and you lose gravitational potential energy.
We can show this as:
Work done against gravity, $W = \Delta U = U_f - U_i$
Work done by gravity, $W = -\Delta U = -(U_f - U_i)$
Where $U_f$ Final potential energy and $U_i$ initial potential energy

One of the possible outcomes of doing work on a system is that the system changes its speed. In this section, we investigate this situation and introduce our first type of energy that a system can possess, called kinetic energy.

$$W = \Delta K = K_f - K_i$$

The Conservation of mechanical energy is

$$E = \Delta U + \Delta KE$$

$$E = (U_f - U_i) + (K_f - K_i) = 0$$

$(K_f + U_f = K_i + U_i$ (Conservation of mechanical energy)

`Example`

A bullet weighing 20g is moving at a velocity of 500m/s. This bullet strikes a windowpane and passes through it. Now, its velocity is 400m/s. Calculate work done by a bullet when passing through this obstacle.

`Solution`

$m = 20g = 0.02kg$, $v_1 = 500m/s$, $v_2 = 400m/s$

We need to determine the change in kinetic energy in this equation. You know that kinetic energy change

$$W = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$W = \frac{1}{2}m(v_2^2 - v_1^2)$$

$$W = \frac{1}{2} \times 0.2kg(400^2 - 500^2)m^2/s^2 = 0.01(160000 - 250000)kgm^2/s^2$$

$$W = -900J$$

This shows that the bullet lost enegy of 900J ie work done by the bullet.

`Activity 5.4`

1. (a) A boy walks up a hill. His displacement from his starting point is (800, 150) m.How much gravitational potential energy has he gained?
   (b) The boy then walks to a village. The displacement from his starting point is (400, –50) m. How much gravitational potential energy did he lose going from the top of the hill to the village?
   (c) What was the boy's net change in gravitational potential energy from his starting point to the village?

2. A spring has a spring constant of 75 N/m. It is stretched by 20 cm. How much energy is stored in the spring?

3. A force of 40 N is used to stretch a spring which has a spring constant of 350 N/m. How much energy is stored in the spring?

4. A spring has a spring constant of 150 N/m and a mass is 100 g is attached to it. The spring sits on a horizontal frictionless surface and the other end of the spring is attached to a solid block. The mass is pulled by 10 cm to stretch the spring and then let go. What is the highest velocity of the mass?

## 5.3 Work energy theorem

According to Newton‘s second law of motion, the sum of all the forces acting on a particle,

$$F = ma$$

Let a force F is applied on an object initially moving with velocity u. If it is displaced to a displacement s and changes its velocity into v, then its motion will be expressed by the equation of motion. We can use the equation of motion

$$v^2 - v_0^2 = 2as$$

Multiplying this equation by mass m and dividing throughout by 2, we get:

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = mas = Fs = W$$

$$\therefore W = \Delta kE$$

`Example`

A car with a mass of 1,000 kg brakes to a stop from a velocity of 20 m/s (45 mi/hr) over a length of 50 meters. What is the force applied to the car? Solution

$$\Delta KE = 0 - [(1/2)(1,000\ kg)(20\ m/s)2] = -200,000\ J$$

$$W = -200,000\ Nm = (F)(50\ m) \Rightarrow F = -4,000\ N$$

`Activity 5.5`

1. A football of mass 550 g is at rest on the ground. The football is kicked with a force of 108 N. The footballer’s boot is in contact with the ball for 0.3 m.

   a) What is the kinetic energy of the ball?

   b) What is the ball’s velocity at the moment it loses contact with the footballer’s boot?

2. A car of mass 1200 kg accelerates from 5 m/s to 15 m/s. The force of the engine acting on the car is 6000 N. Over what distance did the force act?

## 5.4 Conservative forces

Conservative forces a force that does not work when a body moves on a closed path. Conservative forces have these two equivalent properties:

1 The work done by a conservative force on a particle moving between any two points is independent of the path taken by the particle.

2 The work done by a conservative force on a particle moving through any closed path is zero. (A closed path is one in which the beginning and end points are identical.) The gravitational force is one example of a conservative force, and the force that a spring exerts on any object attached to the spring is another.

`Activity 5.5`

| 1. What are the differences between conservative and dissipative forces? |
|---|

## 5.5 Power

Power is the rate at which work is done, or the work done per second. It is measured in the units joules per second (j/s), which are also called watts (W).

$$Power = \frac{total\ \ work\ \ done}{total\ \ time\ \ taken}$$

`Example`:

A garage hoist lifts a truck up 2 meters above the ground in 15 seconds. Find the power delivered to the truck. [Given: 1000 kg as the mass of the truck] `Solution`
First we need to calculate the work done, which requires the force necessary to lift the truck against gravity:
F = mg = 1000 x 9.81 = 9810 N.
W = Fd = 9810N x 2m = 19620 Nm = 19620 J.
The power is P = W/t = 19620J / 15s = 1308 J/s = 1308 W.

`Activity 5.6`

1. A weightlifter lifts 200 kg through 1.8 m in 2 s.
   a) What is the weightlifter's power?
   b) Why is his actual power likely to be higher than this?

2. A petrol engine raises 200 liters of water in a well from a depth of 7 m in 6 seconds. Show that the power of the engine is about 2330 W.

3. Look at question 1 on page 101. It takes 4 seconds to drag the container up the slope. What is the power?

4. Look at question 2 on page 101. The man takes 12 seconds to drag the box. What is his power?

5. A spring with a spring constant of 275 N/m is stretched 20 cm in 2 seconds.What is the power applied to stretch the spring?

`Review questions`

1. A football of mass 550 g is at rest on the ground. The football is kicked with a force of 108 N. The footballer's boot is in contact with the ball for 0.3 m.
   a. What is the kinetic energy of the ball?
   b. What is the ball's velocity at the moment it loses contact with the footballer's boot?

2. A car of mass 1200 kg accelerates from 5 m/s to 15 m/s. The force of the engine acting on the car is 6000 N. Over what distance did the force act?

3. a) A boy walks up a hill. His displacement from his starting point is (800, 150) m.How much gravitational potential energy has he gained?
   b) The boy then walks to a village. The displacement from his starting point is (400, -50) m. How much gravitational potential energy did he lose going from the top of the hill to the village?
   c) What was the boy's net change in gravitational potential energy from his starting point to the village?

4. A spring has a spring constant of 75 N/m. It is stretched by 20 cm. How much energy is stored in the spring?

5. A force of 40 N is used to stretch a spring which has a spring constant of 350 N/m. How much energy is stored in the spring?

6. A spring has a spring constant of 150 N/m and a mass is 100 g is attached to it. The spring sits on a horizontal frictionless surface and the other end of the spring is attached to a solid block. The mass is pulled by 10 cm to stretch the spring and then let go. What is the highest velocity of the mass?

7. A ball of mass 500 g is kicked into the air at an angle of $45^0$. It reaches a height of 12 m. What was its initial velocity?

8. A pendulum bob has a mass of 1 kg. The length of the pendulum is 2 m. The bob is pulled to one side to an angle of $10^0$ from the vertical.

   a) What is the velocity of the pendulum bob as it swings through its lowest point?

   b) What is the angular velocity of the pendulum bob?

9. A weightlifter lifts 200 kg through 1.8 m in 2 s. a) What is the weightlifter's power?

   b) Why is his actual power likely to be higher than this?

10. A petrol engine raises 200 liters of water in a well from a depth of 7 m in 6 seconds. Show that the power of the engine is about 2330 W.

11. . Look at question 1 on page 101. It takes 4 seconds to drag the container up the slope. What is the power?

12. Look at question 2 on page 101. The man takes 12 seconds to drag the box. What is his power?

13. . A spring with a spring constant of 275 N/m is stretched 20 cm in 2 seconds. What is the power applied to stretch the spring?

14. . How can you derive the work–energy theorem form Newton's second law of motion?

# Chapter 6

# Oscillation and Waves

Objective: - At the end of this unit students should be able to:

- Describe the periodic motion of a vibrating object in qualitative terms, and analyse it in quantitative terms (e.g. the motion of a pendulum, a vibrating spring, a tuning fork).
- Define simple harmonic motion (SHM) and describe the relationship between SHM and circular motion.
- Derive and use expressions for the frequency, periodic time, displacement, velocity and acceleration of objects performing SHM.
- Describe the effects: free oscillations, damping, forced oscillations and resonance.
- Explain the energy changes that occur when a body performs SHM.
- Relate the energy of an oscillator to its amplitude.
- Solve problems on SHM involving period of vibration and energy transfer.
- Describe the characteristics of a mechanical wave and identify that the speed of the wave depends on the nature of the medium.
- Calculate the frequency of the harmonics along a string, an open pipe and a pipe closed at one end.

## 6.1 Oscillatory motion

Simple harmonic motion the periodic oscillation of an object about an equilibrium position, such that its acceleration is always directly proportional in size but opposite in direction to its displacement oscillating (vibrating) about a central position equilibrium position the position of an oscillating object when at rest restoring force the force on a displaced object that acts towards its original position.

When a body repeats its path of motion back and forth about the equilibrium or mean position, the motion is said to be periodic. All periodic motions need not be back and forth like the motion of the earth about the sun, which is periodic but not vibratory in nature. The periodic motion in which there is existence of a restoring force and the body moves along the same path to and fro about a definite point, equilibrium position, is called oscillatory motion

### 6.1.1 Harmonic Motion

Any motion that repeats at regular intervals is called periodic motion or harmonic motion. However, here we are interested in a particular type of periodic motion called simple harmonic motion (SHM).

#### Periodic oscillations

If something is oscillating (vibrating) this means that it is moving backwards and forwards, up and down, side to side, in and out, etc, around some central position. This central position is called the equilibrium position and it is the position of the object when it is at rest.

Whenever an object is displaced from its equilibrium position there is a force that acts towards its original position. This force is often referred to as a restoring force, as it tries to restore the system to its equilibrium position. This is much easier to understand if we look at some simple examples.

#### How does a pendulum work?

A simple pendulum is made by hanging a mass, known as the bob, on a string from a fixed support, as shown in Figure 6.1. If we let the mass hang without swinging, it will hang directly below the support with all forces on it balanced. This position, where the resultant force acting on the bob is zero, is known as the equilibrium position.

If we give the bob a small initial displacement by pulling it to one side and then release it, there will be a resultant force, due to the weight of the bob and the tension acting in the string. This force pulls it back towards the equilibrium position. This causes accelerationtowards the equilibrium position (opposite to the direction of displacement).

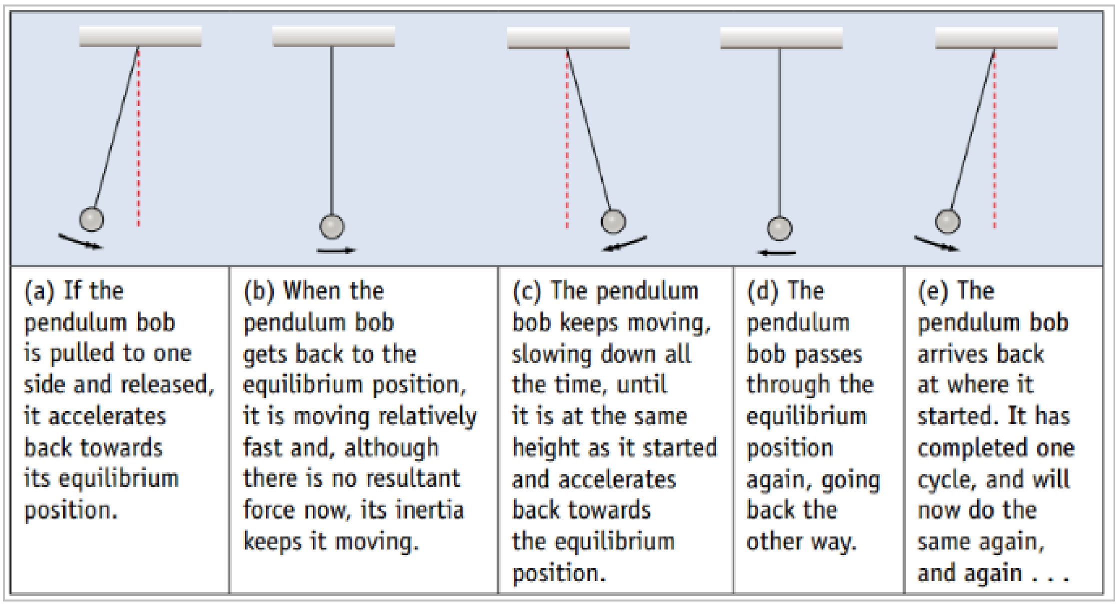



Figure 6.1: *Oscillation of a pendulum when the bob is pulled to one side and released*

When the bob reaches the equilibrium position, the resultant force is now zero, but the bob is moving and can't stop instantly. Its inertia keeps it moving through the equilibrium position, and if there is no significant friction of air resistance, it will keep moving, slowing down all the time until it is as high as it was when it started.

It now has a displacement equal and opposite to its starting displacement. However, as displacement is a vector quantity it is now a negative value. If the initial displacement was 3 cm, the displacement after one swing (half an oscillation) will be –3 cm.

In exactly the same way, it will swing back to where it started to complete one complete cycle of the oscillation. It will now repeat this process again and again. It is important to notice the force causing the oscillation always acts towards the equilibrium position.

### How does a mass on a spring oscillate?

If a mass is hung from a support by a spring and allowed to settle until it is stationary, it will hang with the spring stretched so that the restoring force (in this case the tension in the spring) is equal and opposite to the weight of the mass. This is the equilibrium position.

If we now pull the mass down, the tension in the spring will be greater than the weight of the mass. The resultant force on the mass is upwards and so, if we let go, it accelerates

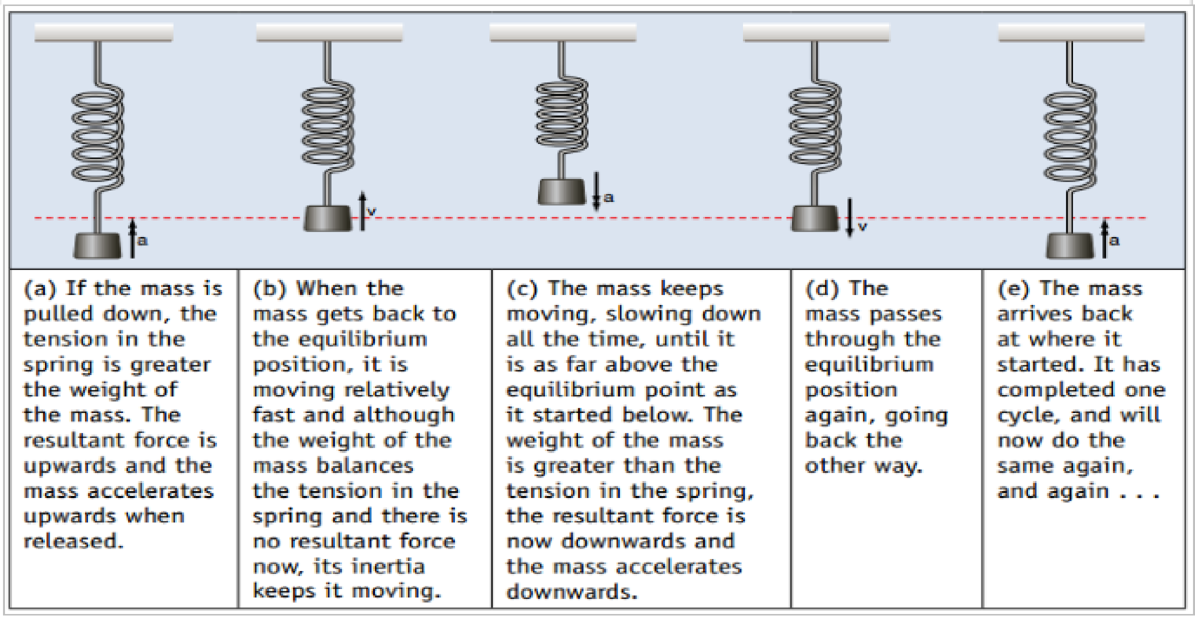



Figure 6.2: *Oscillation of a mass–spring system when the mass is displaced downwards and released*

upwards. When the mass gets back to the equilibrium position it is moving and, although there is no resultant force here, its inertia keeps it moving.

The mass keeps moving, slowing down all the time, until it is as far above the equilibrium point as it started below. The tension in the spring is now less than the weight of the mass, the resultant force is now downwards and the mass accelerates downwards. The mass passes through the equilibrium position again, and carries on until it arrives back at where it started. It has completed one cycle, and will now do the same again, and again.

### What does SHM look like?

If we plot how the displacement of an object performing simple harmonic motion varies with time, we find that the variation is sinusoidal, as shown in Figure 6.3. Note that the displacement goes positive and negative as the mass oscillates either side of the equilibrium position.

The size of the maximum displacement in either direction is called the amplitude A. The time to perform one complete cycle of the oscillation is called the time period T. When we say the oscillation is sinusoidal, we mean that the displacement is described mathematically

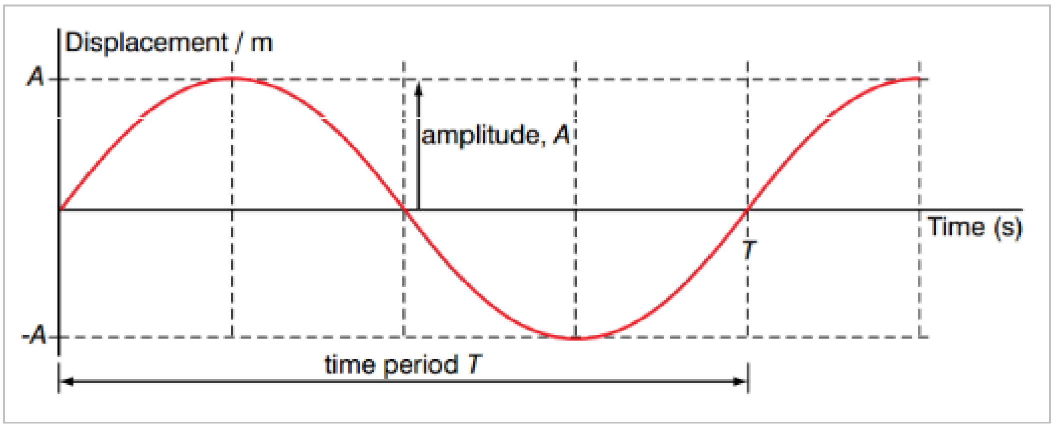



Figure 6.3: *Variation of displacement with time for simple harmonic motion*

using sine or cosine functions:

$$x = Asin\left(2\pi\frac{t}{T}\right) or x = Acos\left(2\pi\frac{t}{T}\right) \tag{6.1}$$

where A is the amplitude of the oscillation and T the time period. Either could be used, but throughout the rest of this chapter we will use,

$$x = Asin\left(2\pi\frac{t}{T}\right) \tag{6.2}$$

although the cosine function gives a better description if the SHM is started by displacing the oscillator and then releasing it.

If $x = Asin(2\pi \cdot \frac{t}{T})$, with $(\frac{2\pi}{T} \cdot t)$ in the expresion relation.

When t = 0,

$$x = Asin\left(\frac{2\pi}{T} \cdot 0\right) = Asin(0) = 0$$

for t= $\frac{T}{4}$

$$x = Asin\left(\frac{2\pi}{T} \cdot \frac{T}{4}\right) = Asin\left(\frac{\pi}{2}\right) = A$$

for t = $\frac{T}{2}$

$$x = Asin\left(\frac{2\pi}{T} \cdot \frac{T}{2}\right) = Asin\left(\pi\right) = 0$$

for t = $\frac{3T}{4}$

$$x = Asin\left(\frac{2\pi}{T} \cdot \frac{3T}{4}\right) = Asin\left(\frac{3\pi}{2}\right) = -A$$

for t = T

$$x = Asin\left(\frac{2\pi}{T} \cdot T\right) = Asin\left(2\pi\right) = 0$$

Looking carefully at the information above you can see how in one oscillation the displacement starts at 0 rises to a positive amplitude, falls back to zero, falls to a negative amplitude and then rises back to zero.

### 6.1.2 Damped and Forced Oscillation

#### Damped Oscillation

The oscillatory motions we have considered so far have been for ideal systems—that is,systems that oscillate indefinitely under the action of only one force a linear restoring force. In many real systems, non conservative forces, such as friction, retard the motion.

Consequently, the mechanical energy of the system diminishes in time, and the motion is said to be damped. Figure 6.4 depicts one such system: an object attached to a spring and submersed in a viscous liquid. One common type of retarding force is proportional to the

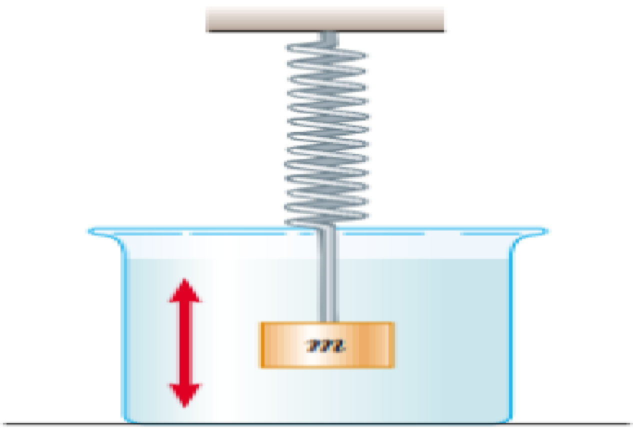



Figure 6.4: *One example of a damped oscillator is an object attached to a spring and submersed in a viscous liquid*

speed of the moving object and acts in the direction opposite the motion. This retarding force is often observed when an object moves through air, for instance.

Air resistance and friction are typical examples of damping forces and are the reason why pendulums naturally stop swinging and masses on springs stop oscillating. The damping force is given by:

$$F_D = -bv \tag{6.3}$$

b = the damping coefficient and is dependent on the medium providing the damping, v = the velocity of the object through the medium.

This equations shows how the resistive force is directly proportional, but opposite, to the velocity. As a result the amplitude of the oscillation will decay exponentially, as shown overleaf in Figure 6.5 (a). Note that the period of the oscillation does not change as the amplitude gets smaller. Heavier damping causes a more rapid decay of amplitude as shown in Figure 6.5(b). Damping in a car suspension is not normally so heavy, as this would produce

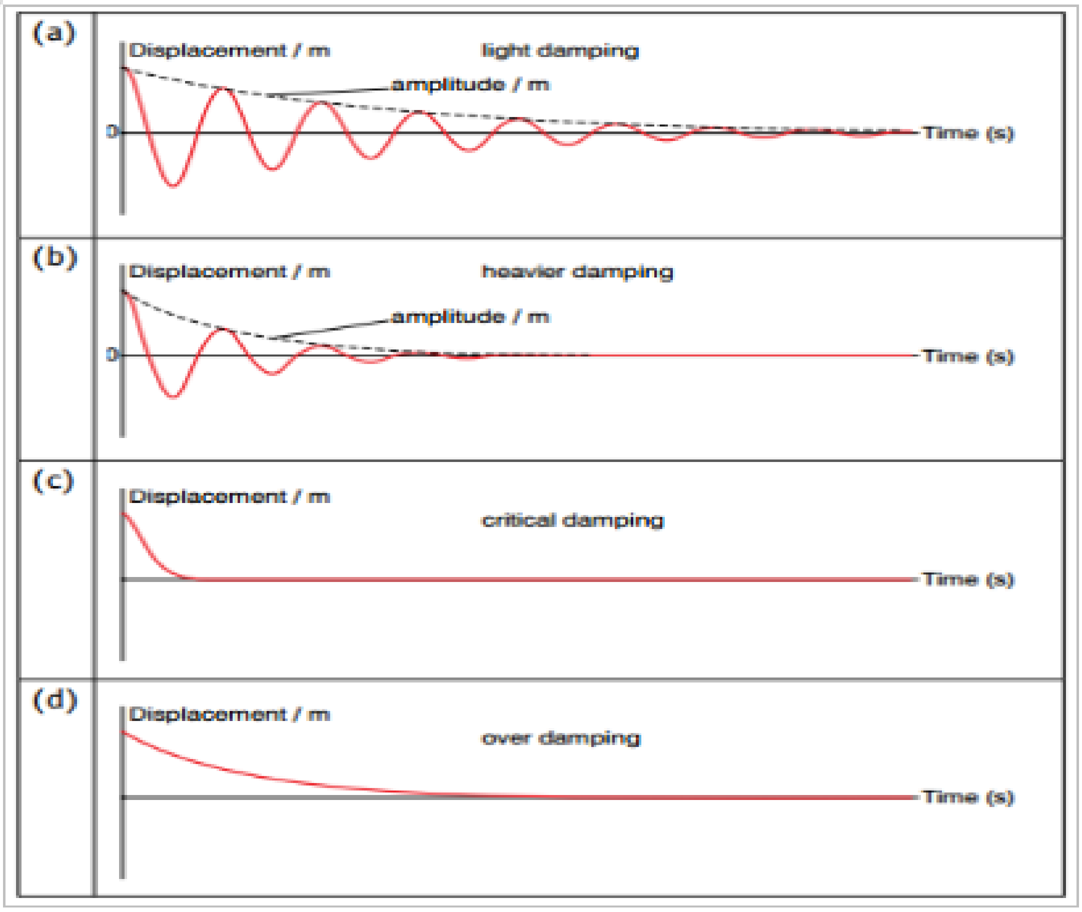



Figure 6.5: *Plots of displacement against time for an oscillator that is displaced and then released, for different amounts of damping.*

a very 'hard' and uncomfortable ride for the passengers. The damping shown in Figure 6.5(b), on the other hand, would provide a very bouncy ride; this would be called **underdamping**.

The damping in a car suspension is always a compromise somewhere near to the critical damping shown in Figure 6.5(c). Critical damping is the amount of damping that leads to the oscillator settling back to a stationary state at the equilibrium position in the shortest possible time.

Damping reduces the effects of resonance. As the periodic driving force transfers energy into the oscillator the damping mechanism dissipates the energy. The resonance peak in the graph of driven amplitude against driving frequency becomes lower and relatively wider, as shown in Figure 6.6. It can also be seen that damping also causes a very small reduction in the natural frequency of the oscillator.

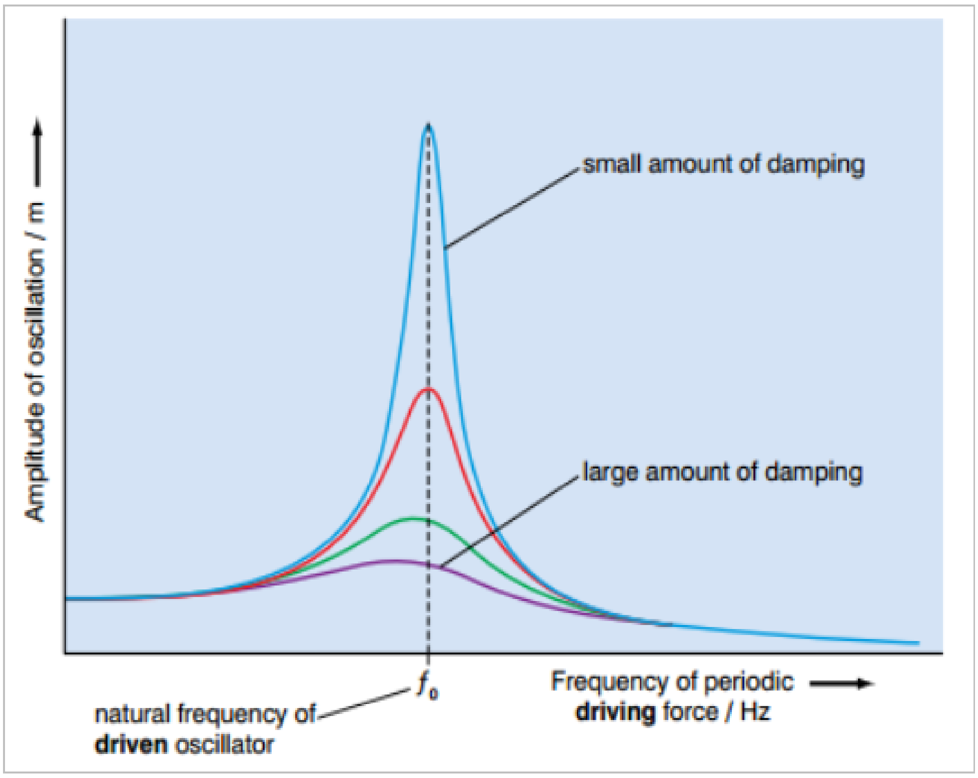



Figure 6.6: *Driven amplitude against driving frequency for forced oscillations of an oscillator with different amounts of damping*

### Forced Oscillation

A common example of a forced oscillator is a damped oscillator driven by an external force that varies periodically, such as $F(t) = F_0 sin\omega t$, where $\omega$ is the angular frequency of the driving force and $F_0$ is a constant. In general, the frequency of the driving force is variable while the natural frequency $\omega$ of the oscillator is fixed by the values of k and m. Newton's second law in this situation gives

$$\sum F = ma \leftrightarrow F_0 sin\omega t - b\frac{dx}{dt} - kx = m\frac{d^2x}{dt^2} \tag{6.4}$$

After a sufficiently long period of time, when the energy input per cycle from the driving force equals the amount of mechanical energy transformed to internal energy for each cycle, a steady-state condition is reached in which the oscillations proceed with constant amplitude.

$$Asin(\omega t + \phi)$$

Where

$$A = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + \left(\frac{b\omega}{m}\right)^2}}$$

and where $\omega_0 = \sqrt{\frac{k}{m}}$ is the natural frequency of the undamped oscillator (b = 0).

For small damping, the amplitude is large when the frequency of the driving force is near the natural frequency of oscillation, or when 0. The dramatic increase in amplitude near the natural frequency is called resonance, and the natural frequency 0 is also called the resonance frequency of the system.

## 6.2 Properties of wave (frequency, wave length, period)

### Terminologies in Wave

**Crests/Troughs**: are positions in a wave with maximum displacements above/below the equilibrium position.

**Amplitude (A)**: is the maximum displacement from the equilibrium position.

**Displacement (y)**: is position of a wave from equilibrium position at any time.

**Wave length** ($\lambda$): distance between any two consecutive points which are in phase.

**Period (T)**: is the time taken by a wave to move one wave length.

**Frequency (f)**: number of oscillations performed per unit time.

**Speed (v)**: is constant in a medium provided the medium is homogeneous.

### What is a travelling wave?

Electromagnetic and sound waves are particularly important to us, but waves on water are a little easier to observe. A travelling wave transfers energy, and sometimes information, from one place to another, in what is called the direction of propagation. An oscillation at the source of energy causes an oscillation to travel through space. For electromagnetic waves this oscillation is of electric and magnetic fields and does not need a medium. In a mechanical wave that involves the oscillations of particles of a physical medium, as the particles pass on energy, they undergo temporary displacements but no permanent change in the position. For example, when ripples travel across a pond the water molecules oscillate vertically but do not move in the direction of the wave.

#### Frequency and time period

The frequency, f, of an oscillation is the number of cycles it completes per second. The unit is the hertz, symbol Hz. A frequency of 50 Hz would correspond to 50 complete oscillations per second. Frequency is related to time period by:

$$f = \frac{1}{T}$$

and so our mathematical expression for displacement can be written as

$$x = Asin(2\pi ft)$$

`Activities`

1. An object moving with simple harmonic motion has an amplitude of 3 cm and a frequency of 30 Hz. Calculate:
2. the time period of the oscillation,
3. the acceleration in the centre and at the maximum displacement of an oscillation, and
4. the velocity in the centre and at the maximum displacement of an oscillation
5. Describe the key features of the different forms of damping the general effect of damping on resonance.

## 6.3 Types of Waves

### 6.3.1 Transverse and longitudinal

Waves can also be categorized as transverse and longitudinal waves based on the way they are propagating.

1. **Transverse Wave**- is a wave where particles of the disturbed medium oscillate perpendicular to the direction of wave motion. Examples are: water waves, waves on strings, and all EM waves. Sinusoidal graphs can represent this motion.

2. **Longitudinal Wave**- is a wave where particles of the disturbed medium oscillate parallel to the direction of wave motion. Example: sound wave

### 6.3.2 Mechanical and Electromagnetic wave

Waves can be categorized as Mechanical and Electromagnetic waves based on the need of material medium for its propagation.

1. **Mechanical Waves**- are waves produced by the oscillation of particles of a mechanical medium and need a medium for propagation. Examples are water waves, sound wave, waves in strings etc.

   All mechanical waves require:

   - some source of disturbance
   - a medium that can be disturbed and
   - physical medium through which elements of the medium can influence each other.

2. **Electromagnetic (EM) waves**:-are produced by accelerated charged particles and can propagate through both material medium and vacuum. Examples are: Light, radio and television waves, micro waves, x-rays, etc. All EM waves in vacuum propagate with speed $c = 3.0 \times 108 m/s$.

Waves can either move in space (e.g water waves), the so called traveling waves, or be stationary in an enclosure, the so called standing waves.

## 6.4 Wave behavior (reflection, refraction, interference, diffraction)

The characteristics of waves are important in determining the size of waves, the speed at which they travel, how they break on shore, and much more. Following are some of the characteristics of waves.

### Reflection of Waves

Whenever a traveling wave reaches a boundary, part or all of the wave bounces back. This phenomenon (rebounding of wave from a surface) is called reflection. For example, consider a pulse traveling on a string that is fixed at one end. When the pulse reaches the wall, it is reflected.

### Refraction of wave

It is the change in direction of a wave passing from one medium to another caused by its change in speed. For example, waves in deep water travel faster than in shallow. If an ocean wave approaches a beach obliquely, the part of the wave farther from the beach will move faster than that closer in, and so the wave will swing around until it moves in a direction perpendicular to the shoreline. The speed of sound waves is greater in warm air than in cold. At night, air is cooled at the surface of a lake, and any sound that travels upward is refracted down by the higher layers of air that still remain warm. Thus, sounds, such as voices and music, can be heard much farther across water at night than in the daytime.

### Diffraction of wave

It is the spreading of waves around obstacles. Diffraction takes place with sound; with electromagnetic radiation, such as light, X-rays, and gamma rays; and with very small moving particles such as atoms, neutrons, and electrons, which show wavelike properties. One consequence of diffraction is that sharp shadows are not produced. The phenomenon is the result of interference (i.e., when waves are superimposed, they may reinforce or cancel each other out) and is most pronounced when the wavelength of the radiation is comparable to the linear dimensions of the obstacle.

### Interference of wave

It is the net effect of the combination of two or more wave trains moving on intersecting or coincident paths. The effect is that of the addition of the amplitudes of the individual waves at each point affected by more than one wave.

Interference also occurs between two wave trains moving in the same direction but having different wavelengths or frequencies. The resultant effect is a complex wave. A pulsating frequency, called a beat, results when the wavelengths are slightly different.

## 6.5 Wave equation

The frequency of a wave can be defined in two equivalent ways. It is the frequency of the individual oscillators that pass the energy along, the number of times particles go up and down or backwards and forwards per second. It is also the number of complete waves, the number of wavelengths that pass any given point per second. If the wavelength is $\lambda$, and f wavelength pass a point per second, then the speed of the wave must be given by the wave equation:

$$v = \lambda f \tag{6.5}$$

The speed of any travelling wave depends on the media it is travelling. For a mechanical wave travelling along a string the speed of the wave depends on the tension of the string and the mass per unit length(sometimes called linear density).

$$v = \sqrt{\frac{T}{\mu}}$$

where $\mu$= mass per unit length given by $\mu$ = m/l in kg/m T = tension in the string in N.

The formula given above shows us that the 'tighter' the string the faster the waves will travel down its length. Additionally the 'lighter' the string, (the smaller its mass/length ratio), the faster the waves will travel down its length. The **phase speed** of a wave is the rate at which the phase of the wave travels through space. Any given phase of the wave (for example, the crest or the trough) will appear to travel at the phase velocity. The phase velocity is given in terms of the wavelength $\lambda$ (lambda) and period T as

$$v_{phase} = \frac{\lambda}{T} \tag{6.6}$$

`Review questions`

1. A simple pendulum is made from a bob of mass 0.040 kg suspended on a light string of length 1.4 m. Keeping the string taut, the pendulum is pulled to one side until it has gained a height of 0.10 m. Calculate

    a) the total energy of the oscillation

    b) the amplitude of the resulting oscillations

    c) the period of the resulting oscillations

    d) the maximum velocity of the bob

e) the maximum kinetic energy of the bob.

2. A piston in a car engine has a mass of 0.75 kg and moves with motion which is approximately simple harmonic. If the amplitude of this oscillation is 10 cm and the maximum safe operating speed of the engine is 6000 revolutions per minute, calculate:

   a) maximum acceleration of the piston

   b) maximum speed of the piston

   c) the maximum force acting on the piston constant?

3. A car of mass 820 kg has an under damped suspension system. When it is driven by a driver of mass 80 kg over a long series of speed bumps 10 m apart at a speed of 3 m/s the car bounces up and down with surprisingly large amplitude.

   a) Explain why this effect occurs.

   b) Calculate the net spring constant of the car suspension system.

4. If you are given a metal rod and a hammer, how must you hit the rod to produce:

   a) a transverse wave, and

   b) a longitudinal wave?

5. A whistle producing a sound at 1 KHz is whirled in a horizontal circle at a speed of 18 m/s. What are the highest and lowest frequencies heard by a listener standing a few metres away, if the speed of sound in air is 340 m/s? 6. If the speed of sound in air is 340 m/s, what is the wavelength of a sound wave at 512 Hz?

6. A travelling wave on a string, of amplitude 2 mm, frequency 500 Hz and speed 300 m/s, can be described by the function

7. $Y = Asin(2\pi - 2\pi ft$

   a) Sketch graphs of displacement Y against distance x for this wave, for the first 1.2 m:

   i) for time t= 0, and

   ii) for time t= 0.5 ms

   b) Sketch graphs of displacement Y against time t for the oscillation produced by this wave for the first 4 ms

i) at the source where x= 0, and

ii) at a distance x= 30 cm from the source.

# Chapter 7

# Heat and thermodynamics

Objective: - By the end of this unit students should be able to:

- Define the zeroth law of thermodynamics.
- Determine the relationship between temperature and energy transfer and thermal equilibrium.
- Give the definitions of isothermal, isobaric, isochoric and adiabatic processes.
- State the first law of thermodynamics.
- Describe ways of changing the internal energy of a gas.
- Solve problems involving calculations of pressure, temperature or volume for a gas undergoing adiabatic changes.
- State the assumptions made to define an ideal gas.
- Describe the kinetic theory of gases, including the importance of Brownian motion and diffusion.

## 7.1 Temperature and Heat

Thermodynamics is a science of the relationship between heat, work, temperature, and energy. Temperature is more difficult to define and we will encounter a number of different ways to approach of temperature.

Temperature is something that we all have experience of. If we place two bodies of different temperatures in contact, then the particles at the boundary will collide and the kinetic energy of particles is transferred backwards and forwards between the objects. A 'body' is another word for an object. On average, the particles in the hotter body have more kinetic energy than those in the colder body, so there is a net transfer of thermal energy from the hotter body to the colder body.

This process is referred to as heating. This is the only way that the word heat can be used. A body does not contain or possess heat. This is just the same as an electrical component, which does not contain or possess electrical current. Instead we will use the term internal energy to describe the total energy that is internal to bodies.

Temperature is a measure of the average random kinetic energy of particles in a body, and is used to determine in which direction there will be a net energy flow when two bodies are close to one another.

### Temperature Scales

Thermometers measure temperature according to well-defined scales of measurement. The three most common temperature scales are Fahrenheit, Celsius, and Kelvin. Temperature scales are created by identifying two reproducible temperatures. The freezing and boiling temperatures of water at standard atmospheric pressure are commonly used. On the Celsius

| | **Absolute Temperature** | **Celsius Temperature** |
|---|---|---|
| Absolute zero | 0.00 | -273.15 |
| Triple point of water | 273.16 | 0.01 |
| Ice point | 273.15 | 0.00 |
| Stean point | 373.15 | 100.00 |
| Room Temperature | 293 | 20 |

scale, the freezing point of water is $0^0C$ and the boiling point is $100^0C$. The unit of temperature on this scale is the degree Celsius ($^0C$). The Fahrenheit scale has the freezing point of water at $32^0F$ and the boiling point at $212^0F$. Its unit is the degree Fahrenheit ($^0F$).

`Example`

"Room temperature" is generally defined in physics to be $25^0C$. (a) What is room temperature in $^0F$? (b) What is it in K?

| To convert from | Use this Equetion |
|---|---|
| Celsius to Fahrenheit | $T_F = \frac{9}{5}T_c + 32$ |
| Fahrenheit to Celsius | $T_c = \frac{5}{9}(T_F - 32)$ |
| Celsius to kelvin | $T_k = T_c + 273.15$ |
| kelvin to Celsius | $T_c = T_k - 273.15$ |
| Kelvin to Fahrenheit | $T_F = \frac{9}{5}(T_k - 273.15) + 32$ |

Solution

To convert from $^0C$ to $^0F$, use the equation

$$T_F = 9/5T_C + 32$$

Substitute the known value into the equation and solve:

$$T_F = 9/5(25^0C) + 32 = 77^0F$$

. Similarly, we find that

$$T_K = T_C + 273.15 = 298K = 298K$$

.

Activity

1 Convert the following to degrees Celsius:

a) the boiling point of helium, 4.25 K

b) the freezing point of gold, 1340 K.

2 Convert the following to kelvin:

a) the freezing point of mercury, -39 $^0$C

b) the average temperature of the universe, -270.42 $^0$C.

## 7.2 The effect of heat on materials (change of Temperature, expansion, change of phase, heat capacity

### Thermal Expansion

The expansion of alcohol in a thermometer is one of many commonly encountered examples of thermal expansion, which is the change in size or volume of a given system as its temperature changes. The most visible example is the expansion of hot air. When air is heated, it expands and becomes less dense than the surrounding air, which then exerts an (upward) force on the hot air and makes steam and smoke rise, hot air balloons float, and so forth.

### Linear Thermal Expansion

The increase in length $\Delta L$ of a solid is proportional to its initial length $L_0$ and the change in its temperature $\Delta T$ .The proportionality constant is called the coefficient of linear expansion, $\alpha$.

$$\Delta L = \alpha L_0 \Delta T \tag{7.1}$$

$$L = L_0 + \Delta L = L_0(1 + \alpha \Delta T) \tag{7.2}$$

$$\alpha = \frac{\Delta L}{L_0 \Delta T}$$

And has unit of $k^{-1}$ or ${}^0c^{-1}$

Table 7.1: **Some typical coefficients of thermal expansion.**

| **Substance** | **Coefficient of Linear expansion, $\alpha(K^{-1})$** |
|---|---|
| Lead | $29 \times 10^{-6}$ |
| Aluminium | $24 \times 10^{-6}$ |
| Brass | $19 \times 10^{-6}$ |
| Copper | $17 \times 10^{-6}$ |
| Iron (steel) | $12 \times 10^{-6}$ |
| Concret | $12 \times 10^{-6}$ |
| Window glass | $11 \times 10^{-6}$ |
| Pyrex glass | $3.3 \times 10^{-6}$ |
| Quartiz | $0.50 \times 10^{-6}$ |

### Areal Expansion

The change in area $\Delta A$ of a solid is proportional to its initial area $A_0$ and thechange in its temperature $\Delta T$ That is,

$$\Delta A = \beta A_0 \Delta T \tag{7.3}$$

$$A = A_0 + \Delta A = A_0(1 + \beta \Delta T) \tag{7.4}$$

Where $\beta = 2\alpha$ is coefficient of arial expansion

### Volume Expansion

The change in volume $\Delta V$ of a solid is proportional to its initial volume $V_0$ and the change in its temperature $\Delta T$. That is:

$$\Delta V = \beta V_0 \Delta T \tag{7.5}$$

$$V = V_0 + \Delta V = V_0(1 + \beta \Delta T) \tag{7.6}$$

Where $\gamma = 3\alpha$ is coefficient of volume expansion

Table 7.2: **Some typical coefficients of thermal expansion.**

| **Substance** | **Coefficient of Volume expansion,** $\gamma(K^{-1})$ |
|---|---|
| Ether | $1.51 \times 10^{-3}$ |
| Carbon Tetrachloride | $1.18 \times 10^{-3}$ |
| Alcohol | $1.01 \times 10^{-3}$ |
| Gasolin | $17 \times 10^{-3}$ |
| Olive Oil | $0.95 \times 10^{-3}$ |
| Water | $0.21 \times 10^{-3}$ |
| Mercury | $0.18 \times 10^{-3}$ |

`Example`

1. A steel rod has a length of exactly 20 cm at $30^0C$. How much longer is it at $50^0C$? [Use $\alpha_{Steel} = 11 \times 10^{-6}/C$.]

   `Solution` The change in temperature of the steel rod is

   $$\Delta T = 50^0C - 20^0C = 30^0C$$

   and the length is 20.0 cm. Using the given value for the linear expansion coefficient $\alpha$, we find the change in length from Equation of expansion,

$$\Delta L = L_0 \alpha \Delta T = (20.0cm)(11 \times 10^{-6}/^0C)(20.0^0) = 4.4 \times 10^{-3}cm$$

The length of the bar increases by $4.4 \times 10^{-3}cm$.

2. By how much does the volume of an aluminum cube 5.00 cm on an edge increase when the cube is heated from $10.0^0Cto60.0^0C$? [Use $\alpha$Al $= 23 \times 10^{-6}/C$]

   `Solution`

   From the given value of $\alpha$ (the linear expansion coefficient) and Equation of areal expansion, we can get $\beta$, the volume expansion coefficient:

$$\beta = 3\alpha = 3(23 \times 10^{-6}/C) = 69 \times 10^{-6}/C$$

   Now, the volume of the cube is $V = (5.00cm)^3 = 125cm^3$; this is really the initial volume, and as usual we don't expect it to change much. The change in temperature of the cube is $\Delta T = 50.0^0C$. the corresponding increase in volume:

$$\Delta V = V\beta\delta T = (125cm^3)(69 \times 10^{-6}/C)(50.0^0C) = 4.3 \times 10^{-1}cm^3 = 0.43cm^3$$

   The volume of the cube increases by $0.43cm^3$

### 7.2.1 Specific Heat and Latent Heat

#### Specific Heats:

Heat flowing into or out of a body (or system) changes the temperature of the body (or system) except during phase changes the temperature remains constant. The quantity of heat, Q, required to change the temperature of a body of mass m by is proportional to both the mass and the change in temperature. Mathematically,

$$Q \sim m\Delta T \Rightarrow Q = mc\Delta T$$

c is a proportionality constant called specific heat capacity(or in short specific heat) of the substance defined as the amount of heat required to raise the temperature of a unit mass of any substance through a unit degree. Its SI unit is $J/kg.KorJ/kg.0^C$. The amount of heat required to change the temperature of n moles of a substance, usually for gases, by $\Delta T$ is :

$$Q = nC\Delta T$$

where C is heat capacity.

**The heat capacity(C)** is defined as the amount of heat energy required to raise the temperature of a substance by $1^0C$.

`Example`

A 0.500 kg aluminum pan on a stove is used to heat 0.250 liters of water from $20.0^0C$ to $80.0^0C$.

(a) How much heat is required? What percentage of the heat is used to raise the temperature of

(b) the pan and (c) the water?

`Solution`

Because water is in thermal contact with the aluminum, the pan and the water are at the same temperature. Calculate the temperature difference:

$$\Delta T \ = \ T_f \ - \ T_i \ = 60.0^0C$$

. Calculate the mass of water. Because the density of water is 1000 kg/m3, one liter of water has a mass of 1 kg, and the mass of 0.250 liters of water is $m_w \ = 0.250kg$. Calculate the heat transferred to the water. Use the specific heat of water $4186J/kg^0C$:

$$Q_w \ = \ mwc_w\Delta T \ = (0.250kg)(4186J/kg^0C)(60.0^0C) = 62.8kJ$$

. Calculate the heat transferred to the aluminum. Use the specific heat for aluminum in Table 1:

$$Q_{Al} \ = \ m_{Al}c_{Al}\Delta T \ = (0.500kg)(900J/kg^0C)(60.0^0C) = 27.0 \times 10^4 \ J = 27.0kJ.$$

Compare the percentage of heat going into the pan versus that going into the water. First, find the total transferred heat:

$$Q_{Total} \ = \ Q_w \ + \ Q_{Al} \ = 62.8kJ + 27.0kJ = 89.8kJ$$

. Thus, the amount of heat going into heating the pan is

$$27.0\ kJ/89.8\ kJ \times 100\% = 30.1\% 27.0\ kJ 89.8\ kJ \times 100\% = 30.1\%$$

and the amount going into heating the water is

$$62.8\ kJ/89.8\ kJ \times 100\% = 69.9\% 62.8\ kJ 89.8\ kJ \times 100\% = 69.9\%.$$

### Latent Heats

Latent Heat the heat required per unit mass of a substance to produce a phase change at constant temperature. The latent heat, $Q_L$ required to change the phase of m mass of a body at constant temperature is calculated as,

$$Q_L = \pm mL$$

Where L is the specific latent heat required to change the phase of 1 kg of a substance at constant temperature.

**Types of Latent Heat Transfer**

There are two types of latent heat transfers between an object and its environment.

**Latent Heat of Fusion ($L_f$):** is the heat absorbed or released when matter melts, changing phase from solid to liquid form at constant temperature. For example, 333.7 kJ of heat is required to change 1 kg of ice to water at $0^0C$, so for water $L_f = 333.7kJ/kg$.

**Latent Heat of Vaporization ($L_V$):** is the heat absorbed or released when matter vaporizes, changing phase from liquid to gas phase at constant temperature. To change 1 kg of water to steam at 1000C, 2256 kJ of heat is required and so $L_V = 2256kJ/kg$.

`Example`

1. If the amount of heat needed for a phase change is 300 kcal, calculate the latent heat of a 5 kg material.

   `Solution:`

   Given parameters are, Q = 300 k.cal M = 5 kg The formula for latent heat is given by,

   $$L = Q/ML = 300/5L = 60k.cal/kg$$

   Hence latent heat value is 60 k.cal/kg

2. At $20^C$, a piece of metal has a density of 60g. When immersed in a steam current at

$100^0C$, 0.5g of the steam condenses on it. Provided that the latent heat of steam is 540 cal/g, calculate the specific heat of the metal.

Solution:

Let c be the specific heat of the metal.

Heat gained by the metal

$$Q = mc\Delta t \Rightarrow Q = 60 \times c \times (100 - 20) \Rightarrow Q = 60 \times c \times 80cal$$

The heat released by the steam

$$Q = m \times LQ = 0.5 \times 540cal$$

By the principle of mixtures, Heat given is equal to Heat taken

$$0.5 \times 540 = 60 \times c \times 80c = 0.056cal/g^0C$$

Hence specific heat value is

$$0.056cal/g^0C$$

Hence, we can conclude that the specific latent heat (L) of a material:

## 7.3 Laws of thermodynamics (zeros, first and second Laws)

### 7.3.1 Zeros Laws of thermodynamics

**The zeroth law of thermodynamics states that:** "Two bodies that are separately in thermal equilibrium with a third body must be in thermal equilibrium with each other." When two bodies are in thermal equilibrium then there is no net transfer of energy between them.F rom our everyday experience, the zeroth law may seem obvious, but it provides us with a way of defining temperature: it is the property of a body that determines whether it is in thermal equilibrium with other bodies. This also enables accurate calibration between thermometers of different kinds.

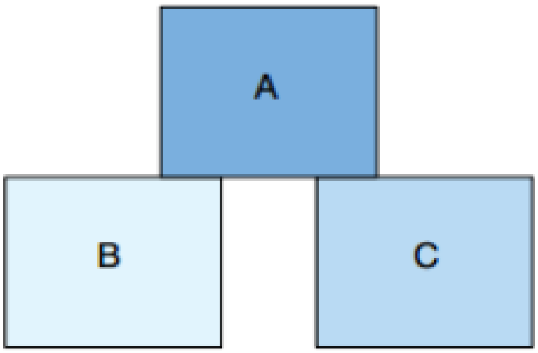



Figure 7.1: *If A is in thermal equilibrium with B, and C is in thermal equilibrium with B, then A is also in thermal equilibrium with C.*

### 7.3.2 First Laws of thermodynamics

The work of Joule mentioned at the start of this section led to the idea that energy as a quantity is conserved whenever any process takes place. This notion is expressed most often as the 'law of conservation of energy', which is a simplification of the first law of thermodynamics. The first law states that: "The increase in internal energy of a system is equal to the sum of the energy entering the system through heating, and the work done on the system." When defining the three quantities, particular attention must be paid to the sign of each quantity. These have the following definitions:
$\Delta U=$ increase in internal energy of the system
$\Delta Q$ = the amount of energy transferred to the system by heating it (that is, by means of a temperature gradient)
$\Delta W$ = the amount of work done on the system
The first law of thermodynamics is therefore written as:

$$\Delta U = \Delta Q + \Delta W \tag{7.7}$$

#### Isochoric process

In a constant volume process, the volume of the system stays constant. Consequently, $W = 0$. From the first law we see that, All the heat entering the system goes into increasing the internal energy.

### Adiabatic Process

In an adiabatic process, the system does not exchange heat with its surroundings; that is, Q = 0. The first law for an adiabatic process takes the form

$$\Delta U = W$$

### Isothermal Process

It is a process which involves no change in the temperature of the system. If the process occurs at constant temperature then there is no change in the internal energy of the system so . The first law for an isothermal process takes the form

$$\Delta U = Q + W$$

$$0 = Q + W$$

$$Q = -W$$

### Isobaric process

In an isobaric process the expansion or compression occurs at constant pressure. Any work done by the system will result in an increase in volume. The work done in Pressure- Volume graph is equal to the area under the PV graph. For an isobaric process the work done W is calculated as

$$W = P\Delta V = P(V_f - V_i)$$

The first law for an isobaric process can be written as

$$\Delta U = Q + W \;\; or \;\; Q - P\Delta V = Q - P(V_f - V_i)$$

### Entropy and the second law of thermodynamics

The second law of thermodynamics states that: "No process is possible in which there is an overall decrease in the entropy of the universe."

`Review of unit questions`

Table 7.3: **Summary of some thermodynamic processes**

| **Process** | **Meaning** |
|---|---|
| Adiabatic | No heat transfer (heating or cooling result from pressure change - Work is either done on or by the gas |
| Isobaric | Constant volume (also called isometric) |
| Isothermal | Constant Temperature |
| Isobaric | Constant Pressure |

1. Explain what is meant by internal energy. Hence suggest how the internal energy of a real gas differs from that of an ideal gas.

2. A heat engine operating between $100^{C}$ and $700^{0}C$ has efficiency equal to 40% of the maximum theoretical efficiency. How much energy does this engine extract from the hot reservoir in order to do 5000 J of mechanical work?

# Chapter 8

# Electrostatics

Learning objectives

At the end of the unit, students will be able to

- State Coulomb's law and solve problems based on it
- Define an electric field and calculate it due to point charges,
- Distinguish between the direction of the Electric Field of positive and negative charges
- Draw Electric Field Lines
- Discuss the Electrostatics field of the conductor
- Define electric potential and electric potential energy
- Derive an expression for the potential at appoint p at a distance r from the charge
- Find the potential difference between the two points
- calculate capacitance

## 8.1 Coulomb's law

Coulombs law gives a relation between two charges $Q_1$ and $Q_2$ which are at a separation r apart. Experiments show that the forces between two bodies obey an inverse square law and that the Force is proportional to the product of the charges. Simply, Coulomb's law states

The Force between two charges at a distance, r apart, is directly proportional to the product of the two charges and inversely proportional to the square of the distance between them Mathematically this is written as

$$F = K\frac{Q_1Q_2}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{Q_1Q_2}{r^2}$$

Where K= $\frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 Nm^2/C^2$ = a constant and is the permittivity of free space. $\epsilon_0$ is a constant called the permittivity of free space (or vacuum permittivity). It has a value of $8.85 \times 10^{-12} F/m$. This constant is fundamental to the study of electric fields. It links electrical concepts such as electric charge to mechanical quantities such as length.

Along with the permittivity of free space, there is a similar constant relating to magnetic fields. This is called the permeability of free space ($\mu_0$).

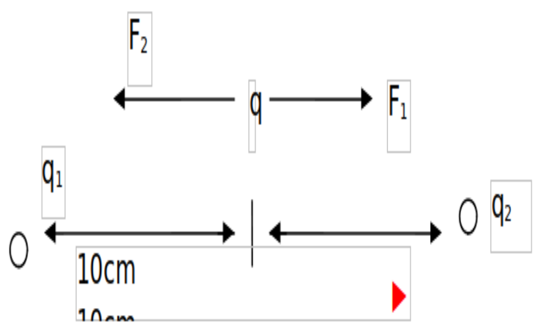
The Force between similar charges is repulsive, and the Force between unlike charges is attractive. In the case of gravitational Force, we can have only attractive Force due to masses. When two charges exert forces simultaneously

on a third charge, the total Force acting on that charge is the vector sum of the forces that the two charges would exert individually. This important property, called the principle of superposing, holds for any number of charges.

$$F = F_1 + F_2 + F_3 + - - - - - - - - - F_n$$

`Example`

A test charge of $q = +1 \times 10^{-6}c$ is placed halfway between a charge of $q_1 = +5 \times 10^{-6}c$ and a charge of $q_2 = +3 \times 10^{-6}c$ that are 20cm apart in the figure below. Find the magnitude and direction of the Force on the test charge `Solution`



The Force exerted on the test charge

$$F_1 = K\frac{qq_1}{r_1^2} = (9 \times 10^9 Nm^2/c^2)\frac{(-1 \times 10^{-6}c)(+5 \times 10^{-6}c)}{(0.1)^2} = +4.5N$$

This Force is to the right and taken as positive. The Force exerted by the charge $q_2$ on q is

$$F_2 = K\frac{qq_2}{r_2^2} = (9 \times 10^9 Nm^2/c^2)\frac{(1 \times 10^{-6}c)(+3 \times 10^{-6}c)}{(0.1)^2} = +2.7N$$

This Force is to the left. If the right is taken as positive, $F_2$ is taken as negative

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 = 4.5N - 2.7N = 1.8N$$

and it acts to the right, that is, towards the $+3 \times 10^{-6}c$ charge

`Exercise`

1. Two charges, one of $+5 \times 10^{-7}c$ and the other $-2 \times 10^{-7}c$ attract each other with a force of 100N. How far apart are they?

2. 3c and 5c charges are separated by 2m. Where between these charges is a third charge placed, in order for the net Force on it to be zero?

3. Three identical charges of $2\mu c$ are placed at (-3, 0) m, (3, 0) m, and (0, 4) m in a rectangular coordinate system. What is the resultant Force on the charge that is placed at (0, 4) m

## 8.2 Electric Field (E)

The concept of an electric field is used to visualize how a charge, or a collection of charges, influences the region around it. The electric field E is analogous to g, which we call the acceleration due to gravity, but which is the gravitational field. Everything we learned about gravity, and how masses respond to gravitational forces can help us understand how electric charges respond to electric forces.

The electric field concept arose to explain action-at-a-distance forces. All charged objects create an electric field that extends outward into the surrounding space. The charge alters that space, causing any other charged thing that enters the space to be affected by this field. The strength of the electric field is dependent upon how charged the object creating the field and upon the distance of separation from the charged objects

### 8.2.1 Electric Field Intensity

Electric field strength is a vector quantity; it has both magnitude and direction. The magnitude of the electric field strength is defined in terms of how it is measured. Let's suppose that an electric charge can be denoted by the symbol Q. This electric charge creates an electric field; since Q is the source of the electric field, we will refer to it as the source charge. The strength of the source charge's electric field could be measured by any other charge placed somewhere in its surroundings. The charge used to measure the electric field strength is referred to as a test charge since it is used to test the field strength. The test charge has a quantity of charge denoted by the symbol q. When placed within the electric field, the test charge will experience an electric force that is either attractive or repulsive. As is usually the case, this Force will be denoted by the symbol F. The electric field's magnitude is defined as the Force per charge on the test charge.

$$Electric\ \ Field = \frac{Force}{Charge}$$

If the symbol E denotes the electric field strength, then the equation can be rewritten in symbolic form as

$$\vec{E} = \frac{\vec{F}}{q}\hat{r}$$

Where $\hat{r}$ is a unit vector

`The Electric field` $\vec{E}$ `at a point in space is defined as the electric force` $\vec{F}$ `acting on a positive test charge q placed at the point divided by the test charge`

The standard metric units of electric field strength arise from its definition. Since the electric field is defined as a force per charge, its units would be force units divided by charge units. In this case, the standard metric unit is Newton/Coulomb (N/C)

The electric field strength is not dependent upon the quantity of the test charge. Now we will investigate a new equation that defines electric field strength in terms of the variables which affect the electric field strength. To do so, we will have to revisit the Coulomb's Law equation When applied to our two charges - the source charge (Q) and the test charge (q).

$$\vec{F} = K\frac{qQ}{r^2}\hat{r}$$

The formula for electric Force can be written as A new equation can be derived if the expression for electric Force given by Coulomb's law is substituted for Force in the above E = F/q equation.

$$\vec{E} = K\frac{Q}{r^2}\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}$$

The electric field strength depends upon the quantity of charge on the source charge Q and the distance of separation r from the source charge.

The strength of an electric field created by source charge Q is inversely related to the square of the distance from the source. This is known as an inverse square law.

Electric field strength is location dependent, and its magnitude decreases as the distance from a location to the source increases. By whatever factor the distance changes, the electric field strength will change inversely by the square of that factor

If a number of point charges $Q_1, Q_2, Q_3, \ldots . Q_n$ are at a distance $r_1, r_2, r_3, \ldots r_n$ from a given point P. each exerts a force on a test charge q placed, and the resultant Force on the test charge is the vector of some of these forces.

$$\vec{E} = E_1 + E_2 + E_3 + ...E_n = \sum_{i=1}^{n} \vec{E}_i$$

Because each term to be summed is a vector, the sum is a vector sum. The fact that the fields that would be caused by the individuals charge is a direct result of the principle of super position.

`Activity 8 .1`

Explain what happens to the magnitude of the electric field created by a point charge as r approaches zero

`Activity 8 .2`

Consider two equal positive or negative point charges separated by the distance d. At what point (other than) would a third test charge experience no net force?

`Example`

1 What are the magnitude and direction of the electric field 1.5cm from a fixed point charge of $+1.2 \times 10^{-10}C$? `Solution` The magnitude of the electric field is computed from

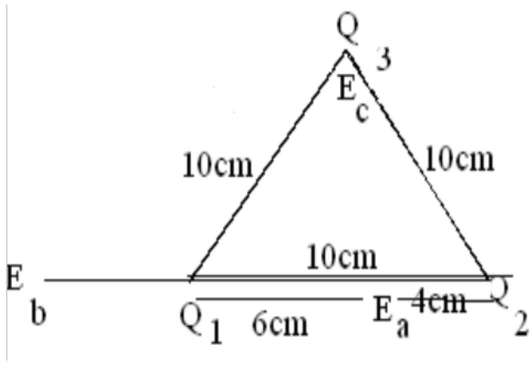
$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r} = K\frac{Q}{r^2}\hat{r} = 9 \times 10^9 Nm^2/c^2 \times \frac{+1.2 \times 10^{-10}C}{(0.015m)^2}$$

$$\vec{E} = 4.8 \times 10^3 N/c$$

- Notice that r was expressed in the SI unit of meters

- The direction of the field is outward from the point charge because the charge is positive

1 Point charge $Q_1$ and $Q_2$ of $12 \times 10^{-9}C$ and $-12 \times 10^{-9}C$ respectively, are placed 0.1m apart as shown Compute the electric fields due to the charge at point a, b, and c




**Solution:** At point a, the vector due to the positive charge Q, is directed toward the right, And the magnitude is

$$\vec{E}_1 = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r} = 9 \times 10^9 Nm^2/c^2 \times \frac{12 \times 10^{-9}C}{(0.06m)^2} = 3 \times 10^4 N/c \ to \ the \ right$$

The vector due to the negative charge $Q_2$ is toward the right, And the magnitude is

$$\vec{E}_2 = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r} = 9\times10^9 Nm^2/c^2\times\frac{12 \times 10^{-9}C}{(0.04m)^2} = 6.75\times10^4 N/c \ to \ the \ right \ to \ the \ right$$

Hence at point a

$$\vec{E} = (3 + 6.75) \times 10^4 N/c = 9.75 \times 10^4 N/c$$

At point b

The vector due to $q_1$ , is directed toward the left, with magnitude

$$\vec{E}_1 = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r} = 9 \times 10^9 Nm^2/c^2 \times \frac{12 \times 10^{-9}C}{(0.04m)^2} = 6.75 \times 10^4 N/c \ to \ the \ right$$

The vector due to the negative charge $Q_2$ is toward the right, And the magnitude is

$$\vec{E}_2 = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r} = 9\times10^9 Nm^2/c^2\times\frac{12 \times 10^{-9}C}{(0.14m)^2} = 0.55\times10^4 N/c \ to \ the \ right \ to \ the \ left$$

Hence at point b

$$\vec{E} = (6.75 - 0.55) \times 10^4 N/c = 9.75 \times 10^4 N/c \ to \ the \ left$$

At point C, the magnitude of each vector is

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r} = 9 \times 10^9 Nm^2/c^2 \times \frac{12 \times 10^{-9}C}{(0.10m)^2} = 1.08 \times 10^4 N/c$$

$$\vec{E_x} = \left(K\frac{Q_1}{r^2}cos(60^0) - K\frac{Q_2}{r^2}cos(60^0)\right)$$

$$= \frac{2KQ}{r^2}cos(60^0$$

$$= \frac{2 \times 9 \times 10^9 \times 12 \times 10^{-9}}{(0.1)^2} \times 0.5 N/c = 1.08 \times 10^4 N/c \ to \ the \ right$$

$$\vec{E_y} = (E_1)_y + (E_2)_y = 0$$

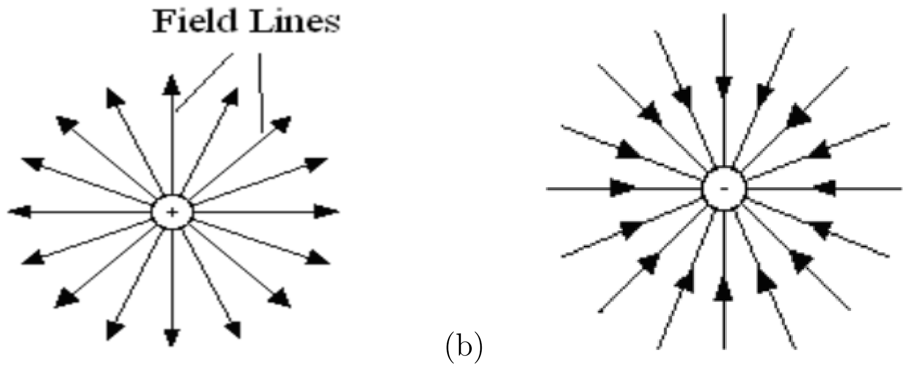
$$\vec{E} = E_a + E_b + E_c = 9.75 \times 10^4 N/c - 6.20 \times 10^4 N/c + 1.08 N/c = 4.63 \times 10^4 N/c \ to \ the \ right$$

## 8.3 Electric Field Lines

A more useful means of visually representing the vector nature of an electric field is through the use of electric field lines of Force. These patterns of lines, sometimes referred to as electric field lines, point in the direction which a positive test charge would accelerate if placed upon the line. As such, the lines are directed away from positively charged source charges, and toward negatively charged source charges.

The electric field can be represented graphically by field lines. These lines are drawn in such a way that, at a given point, the tangent of the line has the direction of the electric field at that point. The density of lines is proportional to the magnitude of the electric field. Each field line starts on a positive point charge and ends on a negative point charge. Since the density of field lines is proportional to the strength of the electric field, the number of lines emerging from a positive charge must also be proportional to the charge.

Electric field lines provide a means to visualize the electric field. Since the electric field is a vector, electric field lines have arrows showing the direction of the electric field. Lines of Force are also called field lines. The direction of the field line at a point tells you what direction the Force experienced by a charge will be if the charge is placed at that point. If the charge is positive, it will experience a force in the same direction as the field; if it is negative the Force will be opposite to the field. The density of lines surrounding any given source




(a) (b)

Figure 8.1: *Electric field from an isolated, (a) Posative Charge (b) Negative Charge*

charge is proportional to the quantity of a charge on that source charge. If the quantity of charge on a source charge is not identical, the pattern will take on an asymmetric nature as one of the source charges will have a greater ability to alter the electrical nature of the surrounding space.

There are a number of principles which will assist in such predictions .These principles are

- Electric field lines always extend from a positively charged object to a negatively charged object, from a positively charged object to infinity, or from infinity to a negatively charged object
- Electric field lines never cross each other
- Electric field lines are most dense around objects with the greatest amount of charge.
- At locations where electric field lines meet the surface of an object, the lines are perpendicular to the surface.

`Activity 8.3`

`A charge 4q is at a distance r from a charge -q. Compare the number of electric field Lines leaving the charge 4q with the number entering the charge -q where do the extra lines beginning on 4q end.`

`Activity 8.4`

`A test charge is released in the field due to two point charges. Do the field lines indicate the possible path traveled by the test charge?`

### Conductors in Electrostatic Fields

In general a conductor can be defined as a region in space where charges are free to move (e.g. a metal). In a static situation the charges don't move. This implies that there is no field within then conductor. Thus, inside a conductor:

$$E(r) = 0$$

A conductor is in electrostatic equilibrium when the charge distribution (the way the charge is distributed over the conductor) is fixed. Basically, when you charge a conductor the charge spreads itself out. At equilibrium, the charge and electric field follow these guidelines:

- the excess charge lies only at the surface of the conductor
- the electric field is zero within the solid part of the conductor
- the electric field at the surface of the conductor is perpendicular to the surface
- charge accumulates, and the field is strongest, on pointy parts of the conductor

## 8.4 Electric potential of a point charge

The electric potential of a point charge is

$$V = kQ/r$$

. where k is a constant equal to $9.0 \times 10^9 Nm^2/C^2$. Electric field is a vector while electric potential is a scalar. The voltage resulting from a combination of point charges is obtained by adding voltages as integers, whereas the overall electric field is obtained by adding individual fields as vectors.

Point charges, such as electrons, are among the fundamental building blocks of matter. Furthermore, spherical charge distributions (like on a metal sphere) create external electric fields exactly like a point charge. The electric potential due to a point charge is, thus, a case we need to consider. Using calculus to find the work needed to move a test charge q from a large distance away to a distance of r from a point charge Q , and noting the connection between work and potential

$$W = -q\Delta V$$

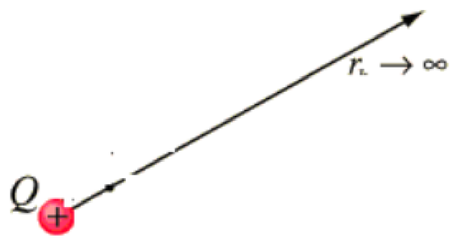
, we can define the electric potential V of a point charge:

### Electric potential

Consider a charge q placed in an electric field E. Let us chose some arbitrary reference point A in the field at this point the electric potential energy of the Charge is defined be zero. This defines the electric potential energy of the charge at every other point in the field. For instance, the electric potential energy UB at some point B is simply the work W done in moving the charge from A to B along any path: It is clear that depends on both the particular charge q which we place in the field and the magnitude and direction of the electric field along some arbitrary route between points A and B. We can exploit this fact to define a quantity known as the electric potential. The difference in electric potential between two points B and A in an electric field is simply the work done in moving some charge between the two points divided by the magnitude of the charge. Thus,

$$V_B - V_A = \frac{\Delta W}{q} = \frac{\Delta U}{q}$$

The general expression for the electrical potential of a point charge Q can be obtained by referencing to a zero of potential at infinity. The expression for the potential difference then.




$r_B$ goese to infinity is gives simply

$$V = \frac{KQ}{r} = \frac{Q}{4\pi\epsilon_0 r}$$

The zero of electric potential (voltage) is set for convenience, but there is usually some physical or geometric logic to the choice of the zero point. For a single point charge or localized collection of charges, it is logical to set the zero point at infinity. If there are n number of charges in space , the potential at apoint is found by superposition principles that is the electric potential due to a number of charges is the algebraic sum of the individuals potentials. The total electric potential point p is the sum of the potential due to charges

$$Q_1, Q_2, Q_3, ..., Q_n$$

$$V = \frac{KQ_1}{r_1} + \frac{KQ_2}{r_2} + ... + \frac{KQ_n}{r_n} = \sum_{i=1}^{n} \frac{Q_i}{r_i}$$

Note; potential a scalar quantity

`Activity 8.5`

> In a certain region of space the electric field is zero From this we can conclude that the electric potential in this region is A) zero B) constant C) positive D) negative

The dimensions of electric potential are work (or energy) per unit charge. The units of electric potential are, therefore, joules per Coulomb (J/c). A joule per Coulomb is usually referred to as a volt (V)

$$1J/c = 1V$$

Consider a charge q which is slowly moved a small distance +x along the x-axis. Suppose that the difference between the electric potential at the final and initial positions of the charge is +V. By definition, the change +U in the charge's electric potential energy is given by

$$+U = q + V = W$$

$$q\Delta V = qE\Delta r$$

$$\Delta V = E\Delta r$$

$$E = \frac{\Delta V}{\Delta r}$$

Where E is the electric field strength According to equation electric field strength has the dimension of potential difference over the length. It follows that the unit of electric field is volt per mete (Vm).

### 8.4.1 Motion of charged particles in an electric field

When a particle of charge of and mass m is placed in an electric field E, the electric Force exerted on the charge is qE. If this is the only Force exerted on the particle it must be the net Force and cause the particle to accelerated according to Newton's second law

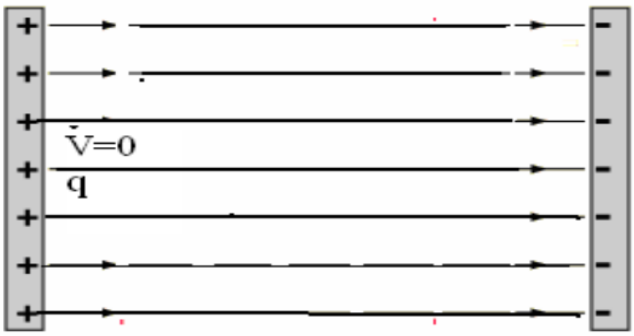
$$F_e = qE = am$$

$$a = \frac{qE}{m}$$

If E is uniform the acceleration is constant. If the particle has a positive charge, its acceleration is in the directing of the electric field. If the particle has a negative charge, its acceleration is in the direction of opposite the electric field.

Example

1. As shown in the figure below, a positive point charge q of mass m is released from rest in a uniform electric field E directed along the x-axis. Describe its motion




Solution

The acceleration a is constant and is given by q E/m. The motion is simple linear motion along the x axis. Therefore we can apply the equation of kinematics in one dimension

$$x_f = x_i + v_i t + \frac{1}{2}at^2$$

$$v_f = v_i + at$$

$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$

Choosing the initial position of the charge as $x_i = 0$ and assigning $V_i = 0$ because the particle starts from rest, the position of the particle as a function of time is

$$x_f = \frac{1}{2}at^2 = \frac{qE}{m}t^2$$

The speed of the particle is given by

$$v_f = \frac{qE}{m}t$$

The third kinematics equation gives us

$$v_f^2 = 2ax_f = \left(\frac{2qE}{m}\right)x_f$$

from which we can find the kinetic energy of the charge after it has moved a distance

$$KE = \frac{1}{2}mv_f^2 = \frac{1}{2}m\left(\frac{2qE}{m}\right)(x_f - x_i) = qE\Delta x$$

2 An electron entrees the region of a uniform electric field as shown with $V_i = 3 \times 10^6$ m/s and E = 200N/C. The horizontal length of the plare, L = 0.100 m

i) Find the acceleration of the electron while it is in the electric field

ii) If the electron enters the field at t = 0 find the time at which it leaves the field

iii) If the vertical position of filed the electron as enters field is $y_i = 0$. What the vertical position when it leaves the fields

`Solution`

i) The charge on the electron has $1.6 \times 10^{-19}C$ and $m_c = 9.11 \times 10^{-31}kg$

$$a = -\frac{qE}{m_e}j = -\left(\frac{1.6 \times 10^{-19}c}{9.11 \times 10^{-31}kg}\right)$$

$$a = -3.51 \times 10^{13}m/s^2$$

ii) The horizontal distance across the field is L = 0.1m. We find that the time at which the electron exists the electric field is

$L = 0.1m$

$$t = \frac{L}{v_i} = \frac{0.1m}{3 \times 10^6 m/s} = 3.33 \times 10^{-8}s$$

iii) Using the results from part A and B we find that

$$y_f = \frac{1}{2}a_y t^2 = -\frac{1}{2}\left(3.51 \times 10^{13}m/s^2\right)\left(3.33 \times 10^{-8}s\right)^2 = -0.0195m = -1.95cm$$

## 8.5 Capacitance and Capacitor networks

A capacitor is a devise that is used to store electric charge. It is usually made up of two plates separated by a thin insulating material known as the dielectric. The capacitance of a system depends only on its shape and on the insulators it contains. One plate of the capacitor is positively charged, while the other has a negative charge. The charge in a capacitor is proportional to the potential difference between the plates. For a capacitor with charge Q on the positive plate and -Q on the negative plate, the capacitance measures the amount of charge a capacitor can store. A convenient measure of the ability of a device to store electric charge is its capacitance C.

A battery will transport charge from one plate to another until the voltage produced by the charge buildup is equal to the battery voltage. The capacitance of an object is defined as being equal to the charge required to raise the potential of that object by one V

$$C = \frac{Q}{V}$$

Or

$$Q = CV$$

Where C is the capacitance in Farad Q is the charge in Coulomb stored in each plate V is the potential in Volts applied to the plate

`The capacitor's capacitance (C) is a measure of the amount of charge (Q) stored on each plate for a given potential difference or voltage (V) which appears between the plates.`

The SI unit of capacitance is the farad F The circuit symbol for a capacitor is 1farad=1Coulomb/1Volt

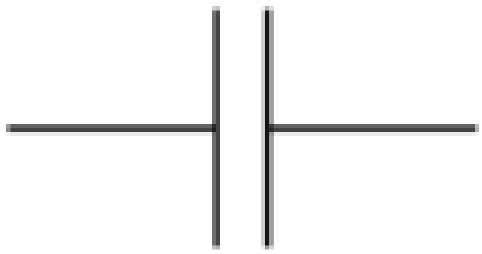


Figure 8.2: *The circuit symbol for a capacitor*

`Activity`

A $25\mu F$ capacitor is charged to a potential of 18V. How much charge stored on capacitor?

## 8.6 The Parallel Plate Capacitor

Consider two large flat plates placed near one another. The plates are parallel, and have equal and opposite charges uniformly distributed. This configuration is known as a parallel-plate capacitor. A parallel-plate capacitor is a great way to create a uniform field.

Consider a capacitance C in vacuum consisting of two parallel plates, each with area A separated by a distance d as shown in figure 1. One plate carries acharge Q, and the other carries a charge-Q. The amount of a charge that can be stored on a plate for a given potential increases as a plate area increased. Thus ,we expect the capacitance to be proportional to the plate area A.

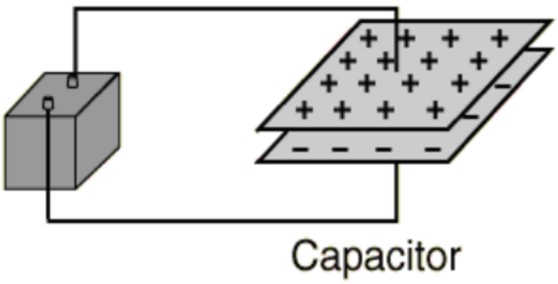



Figure 8.3: *The Parallel Plate Capacitor*

Now consider the region that separates the plate. The electric field between the plates must increase as d decreased. Moving the plates together causes the charge on the capacitor to increases. If d is increased, the charge deceases. As a result, we expect the capacitance of the pair of the plates to be inversely proportional to d. We can verify these physical arguments with the following derivation. The surface charge density on either plate is

$$\delta = \frac{Q}{A}$$

The magnitude of the Electric field has a very simple relation to the voltage between the plates and their separation d.

$$E = \frac{V}{d}$$

Using the definition of capacitance we can determine the capacitance C of an ideal capacitor as a function of its structure.

$$C = \frac{Q}{V} = \frac{Q}{Ed} = \frac{Q}{\frac{\delta}{\epsilon_0}d}$$

$$= \epsilon_0 \frac{Q}{\frac{Q}{A}d}$$

$$C = \epsilon_0 \frac{A}{d}$$

This equation for the capacitance of a parallel capacitor shows that C is a constant independent of the charge stored in on the plates or the voltage across the capacitor.The capacitance of a system depends on its shapes, dimensions and separation of the conductors that make up the capacitor

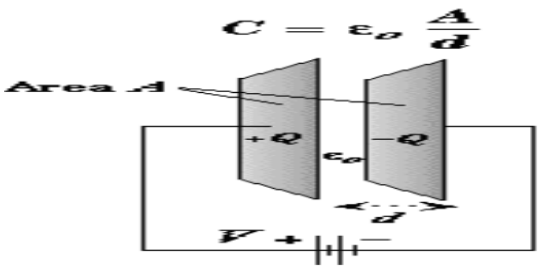



Figure 8.4: *TThe Parallel Plate Capacitor*

`Example`

1) The plates of a parallel-plate capacitor are 5mm apart and 2m 2 in area. The plates are in vacuum. A potential difference 10, 000v is applied across the capacitor compute

   A) The capacitance

   B) The charge on the plate

   C) The electric intensity in the space b/n them

`Solution` A)

$$C = \epsilon_0 \frac{A}{d} = \left( \frac{8.85 \times 10^{-12} c^2/Nm^2}{5 \times 10^{-3} m} \right) (2m^2)$$

$$C = 3.54 \times 10^{-9} C^2/Nm = 3.54 \times 10^{-9} F$$

B) The charge on the plate

$$Q = CV_{ab} = (3.54 \times 10^{-9} C/V)(10^2 V) = 3.54 \times 10^5 c$$

C) The electric intensity in the space b/n them

$$E = \frac{\delta}{d} = \frac{Q}{\epsilon_0 A} = \frac{3.54 \times 10^5 C}{8.85 \times 10^{-12} c^2/Nm(2m^2)}$$

$$= 20 \times 10^5 N/c$$

Since the electric field equals the potential gradient

$$E = \frac{V_{ab}}{d} = \frac{10^4 V}{5 \times 10^{-3} m} = 20 \times 10^5 v/m$$

$$\frac{N}{C} = \frac{V}{m}$$

2 Parallel-plate capacitor is designed to have a capacitance of 1.00F when the plates are separated by 1.00mm in vacuum what must be the arch of the plates. (Ans $A = 1.13 \times 10^8 m^2$)

### 8.6.1 Energy Stored in a Capacitor

The energy stored in a capacitor is the same as the work needed to build up the charge on the plates. As the charge increases, the harder it is to add more. Potential energy is the charge multiplied by the potential, and as the charge builds up the potential does too. If the potential difference between the two plates is V at the end of the process, and 0 (zero) at the start, the average potential is V/2. Multiplying this average potential by the charge gives the potential energy.

$$PE = 1/2QV$$

. Substituting in for Q, Q = CV, gives: The energy stored in a capacitor is: intermes of C and V.

$$W = U = \frac{1}{2} CV^2$$

Substituting Q = CV and $V = \frac{Q}{C}$

$$U = \frac{1}{2} QV$$

This is U intermes of Q and V

$$U = \frac{Q^2}{2C}$$

intermes of Q and C Where U=Electric potential energy in joule Q=Charge in Coulomb. V=Potential in volt C = Capacitance in farad These formulae are valid for any type of capacitor, since the arguments we used to derive them do not depend on any special property of parallel plate capacitors. The potential difference between the plates is V = Ed and $C = \frac{\epsilon_0 A}{d}$ Thus, the energy stored in the capacitor can be written as

$$U = W = \frac{CV^2}{2}$$

$V = Ed$, $C = \frac{\epsilon_0 A}{d}$

$$U = \frac{\epsilon_0 A E^2 d^2}{2d}$$

$$U = \frac{\epsilon_0 A E^2 d}{2}$$

Now, Ad is the volume of the field filled region between the plates, so if the energy is stored in the electric field then the energy per unit volume, or energy density, of the field must be

$$u = \frac{U}{V} = \frac{1}{2}\frac{CV^2}{Ad}$$

Substituting

$$C = \epsilon_0 \frac{A}{D}$$

$$\frac{V^2}{d^2} = E^2$$

$$u = \frac{1}{2}\epsilon_0 \frac{V^2}{d^2} = \frac{1}{2}\epsilon_0 E^2$$

1) Air filled parallel plate capacitor has a capacitance of 5.0 pF. Apotential of 100V is applied across the plates, which are 1.0 cm apart, using astorage battery.

   a) What is the energy stored in the capacitor? Suppose that the battery is disconnected and the plates are moved until they are 2.0 cm apart.

   b) What is the energy stored in the capacitor now?

   c) Suppose, instead, that the battery is left connected and the plates are again moved until they are 2.0 cm apart. What is the energy stored in the capacitor in this case?

`Solution`

The initial energy stored in the capacitor is

$$U = \frac{CV^2}{2} = \frac{5 \times 10^{-12}}{2}(1000)^2 J = 2.58 \times 10^{-8} J$$

When the spacing between the plates is doubled, the capacitance of the cpacitor is halved to 2.5 pF. If the battery is disconnected then this process takes Place at constant charge Q. Thus, it is obvious from the formula

$$U = \frac{Q^2}{2C}$$

That in this case the energy stored in the capacitor doubles. So, the new energy is

$$U = 2(2.58 \times 10^{-8})J$$

$$= 5.16 \times 10^{-8}J$$

## 8.7 Capacitance net work

### Parallel Combination

Capacitors are one of the standard components of electronic circuits. Complicated combinations of capacitors often occur in practical circuits. It is, therefore, useful to have a set of rules for finding the equivalent capacitance of some general arrangement of capacitors. It turns out that we can always find the equivalent capacitance by repeated application of two simple rules. These rules related to capacitors connected in series and in parallel. In a parallel combination, the capacitors are usually drawn side by side. If we imagine them as parallel-plate capacitors with the same gap, snuggling them right up next to each other, the combination seems to become a single capacitor with an area equal to the sum of the areas. Then from the equation for capacitance of a parallel-plate capacitor, we have.

$$C_{eq} = \frac{\epsilon_0 A_{eq}}{d} = \frac{\epsilon_0 (A_1 + A_2)}{d}$$

$$= \frac{\epsilon_0 A_1}{d} + \frac{\epsilon_0 A_2}{d}$$

$$C_{eq} = C_1 + C_2$$

Or consider two capacitors connected in parallel; i.e. with the positively charged Plates connected to a common "input" wire and the negatively charged plates attached to a common "output" wire. What is the equivalent capacitance between the input and output wires? In this case, the potential difference V across the two Capacitors is the same, and is equal to the potential difference between the input and output wires.

The total charge Q, however, stored in the two capacitors is di-vided between the capacitors, since it must distribute itself such that the voltage across the two is the same. Since the capacitors may have different ,C 1 and C 2 , the charges Q 1 and Q 2 may also be different. The equivalent capacitance $C_{eq}$ of the pair of capacitors is simply the ratio Q/V

$$C_{eq} = \frac{Q}{V} = \frac{Q_1}{V} + \frac{Q_2}{V}$$

$$C_{eq} = C_1 + C_2$$

When a number of capacitors are connected in parallel, the total or effective capacitance

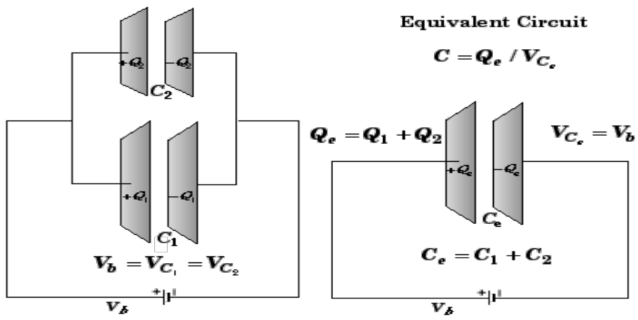



Figure 8.5: *Capacitors connected in Parallel*

of the group is equal to the sum of the individual capacitances The equation for calculating the total capacitance C obtained by capacitances $C_1$ , $C_2, C_3$ etc..The formula for parallel capacitor is same as the resistance in series. The working voltage of parallel capacitors is equal to the lowest working voltage rating in the combination. Parallel connected Capacitors

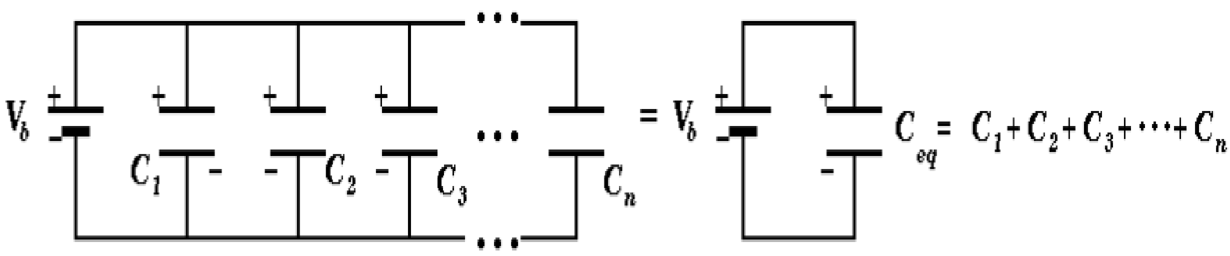



Figure 8.6: *Capacitors connected in Parallel*

always have the same voltage drop across each of them. They do not have the same charge unless they have the same capacitance C. The charge on the equivalent capacitor C eq is the sum of the charges on both capacitors. The Voltage on the equivalent capacitor C eq is the same as the voltage across either capacitor.

`The equivalent capacitance of capacitors connected in parallel is the sum of the individual capacitances`

### Series Combination

In a series combination, the capacitors are connected head-to-tail. We want to replace the pair or more by a single equivalent capacitor. To do this, we must understand how the charge is distributed on the plates.

Consider the inner pair of plates, one from each capacitor, connected by a conductor. These three objects are electrically isolated from the remainder of the circuit; they form a single isolated conductor. Since the net charge on the capacitors is zero before the battery is connected, the net charge on the inner pair of plates must also be zero. After the battery is connected, the plates of the capacitors will hold some charge, but the inner pair of plates will still have zero net charge. Therefore, the charges on the inner pair of plates are equal and opposite, and we see that both capacitors will hold the same charge. We don't add these charges together, as in the parallel case. The quantity that adds is the voltage across each capacitor. consider capacitors arranged so that the potential across the combination is equal to the sum of the potential difference across each as shown in fig

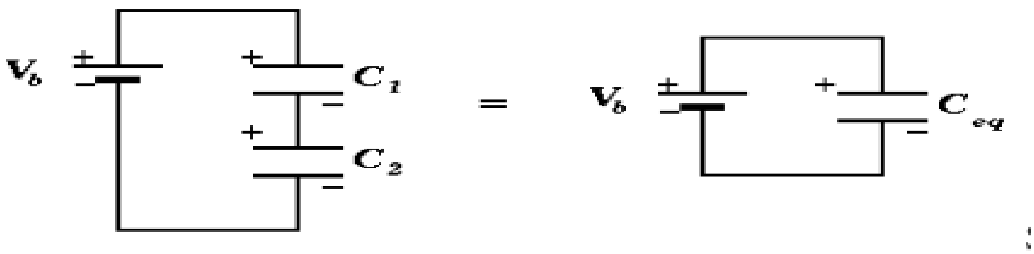



Figure 8.7: *Capacitors connected in Series*

$$\Delta V = \Delta V_1 + \Delta V_2$$

The voltege acrossacrs sthecapacitor is releated totheir ch arg es

$$\Delta V_1 = \frac{Q}{C_1}$$

and

$$\Delta V_2 = \frac{Q}{C_2}$$

The defineton of equvalent capacitor is

$$C_{eq} = \frac{Q}{\Delta V}$$

Or

$$\Delta V = \frac{Q}{C_{eq}}$$

Therfore

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

For more than two Capacitor

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ... + \frac{1}{C_n} = \sum_{i=1}^{n} \frac{1}{C_i}$$

Series connected Capacitors always have the same charge. They do not the same voltage

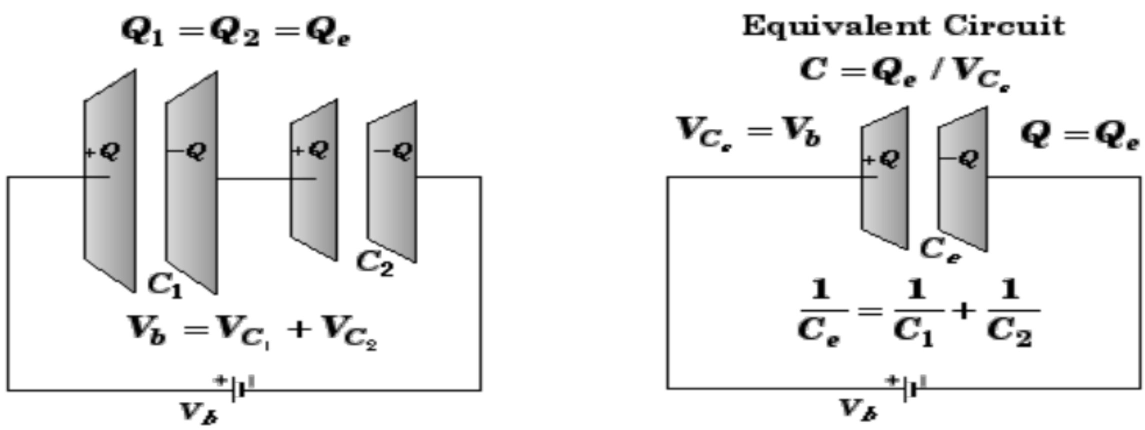



Figure 8.8: *Capacitors connected in Series*

unless the capacitors have the same Capacitance C. The charge on the equivalent capacitor $C_e$ is the same as the charge on either capacitor. The Voltage across the equivalent capacitor $C_eq$ is the sum of the voltage across both capacitors. If two or more capacitors are connected in series as shown above, the total capacitance is less than that of the smallest capacitor in the group

`Example`

1) Let $C_1 = 6\mu F$ and $C_2 = 3\mu F$, $V_{ab} = 18V$

   A. What is the equivalent capacitance of the series combination

   B. What is the charge on each capacitor

   C. Find the potation difference across the capacitor.

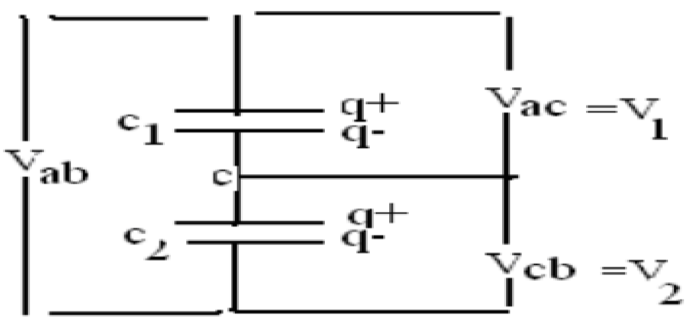



Figure 8.9: *Capacitors connected in Series*

`Solution`

a) for series combination

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$= \frac{1}{6\mu F} + \frac{1}{3\mu F} = \frac{1}{2\mu F}$$

$$C_{eq} = 2\mu F$$

b) The charge Q is

$$Q_1 = Q_2 = C_{eq}V_{ab} = (2\mu F)(18V) = 36\mu C$$

c) The potential difference across the capacitor are

$$V_{ac} = V_1 \frac{Q}{C_1} = \frac{36\mu F}{6\mu F} = 6V$$

$$V_{cb} = V_2 = \frac{Q}{C_2} = \frac{36\mu F}{3\mu F} = 12V$$

1) A 1$\mu$F and a 2$\mu$F capacitor are connected in parallel and this pair of capacitors is then connected in series with a 4$\mu$F capacitor.

   i) What is the equivalent capacitance of the whole combination?

ii) What is the charge on the $4\mu$ F capacitor if the whole combination is connected across the terminals of a 6V battery?

iii) What are the charges on the $1\mu$F and $2\mu$F capacitors?

`Answer`:

The equivalent capacitance of the $1\mu$F and a $2\mu$F capacitors connected in parallel is $1\mu$F + $2\mu$F = $3\mu$F . When a $3\mu$F capacitor is combined in series with $4\mu$F capacitor the equivalent capacitance of the whole combination is given by

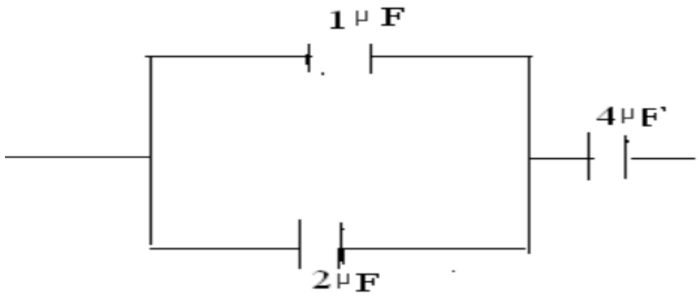



Figure 8.10: *Capacitors connected in Series*

$$\frac{1}{C_{eq}} = \frac{1}{3\mu F} + \frac{1}{4\mu F}$$
$$= \frac{7}{12 \times 10^{-6}} F^{-1}$$
$$C_{eq} = \frac{12 \times 10^{-6}}{7} F = 1.71\mu C$$

The charge delivered by the 6 V battery is Q = $C_{eq}$, $V = (1.71 \times 10^{-6})(6) = 10.3\mu c$. This is the charge on the $4\mu$F capacitor, since one of the terminals of the battery is connected directly to one of the plates of this capacitor. The voltage drop across the $4\mu$F capacitor is

$$V_A = \frac{Q}{C_4} = \frac{10 \times 10^{-6} C}{4 \times 10^{-6} F} = 2.57V$$

Thus, the voltage drop across the $1\mu$F and $2\mu$F combination must be

$$V_{12} = 6V - 2.57V = 3.43$$

The charge stored on the $1\mu$F is given by

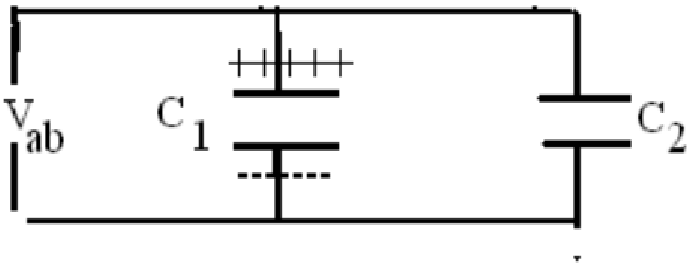
$$Q_1 = C_1 V_{12} = (1\mu F)(3.43) = 3.42\mu F$$

Likewise, the charge stored on the 2$\mu$F capacitor is

$$Q_2 = C_2V_{12} = (2\mu F)(3.43) = 6.84\mu F$$

Note that the total charge stored on the 1$\mu$F and 2$\mu$F combination is

$$Q_{12} = Q_1 + Q_2 = 10.3\mu C$$




In fig $C_1 = 6\mu$F and C 2 $= 3\mu$F and V ab 18V find

a Equivalent capacitance

b The charge on each capacitor

c The potential difference on each capacitor

`Solution`

A) The equivalent capacitance of the parallel combination is $C_{eq} = C_1 + C_2 = 6\mu F + 3\mu F = 9\mu F$ B) The charge $Q_1 and Q_2$ are

$$Q_1 = C_1V = (6\mu F)(18V) = 108\mu C$$

$$Q_2 = C_2V = (3\mu F)(18V) = 54\mu C$$

C) The potential is the same for each capacitor. Because they are connected unparallel

`Solution`

$$C_0 = \epsilon_0 \frac{A}{d} \left(8.85 \times 10^{-12} C^2/Nm^2\right)$$

$$= 17.7 \times 10^{-11} F = 177PF$$

`Unit Exercise`

1) What is the magnitude and direction of the electric field that will balances the weight of?

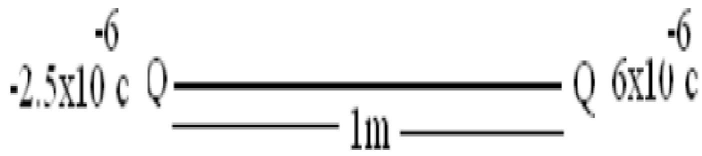
   a) an electron and

   b) a proton

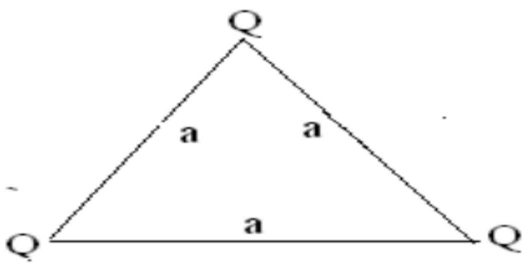
2) In figure determine the point (other than infinity) at which the electric field is zero

3) Two point charges are located on the X axis.The first is a charge to Q at X = - a. The second is an unknown charge located at X = 3a. The net electric field these charges produce at the origin has a magnitude of $\frac{2KQ}{a^2}$ . What are the two possible values of the known charge?

4) Determine the point at which the electric field zero.




5 Three equal positive charges are at the corners of an equilateral triangle of side a as shown.




A Three Charges together create an electric field. Sketch the field lines in the plane of the charge

B Find the location of the point (other than infinity) where the electric field is zero

6 Find the potential at a distance 1cm from a proton (B) What is the potential difference between two points that are 1cm and 2cm from a proton? (c) What if ? Repeat part (a) and (b) for an electron

7 At a certain distance from point charge, the magnitude of the electric field is 500v/m and the electric potential is -3.00kv

a) What is the distance to the charge?

b) What is the magnitude of the charge?8)

8) A proton accelerates from rest in a uniform electric field 640N/C its speed is 1.2x10 6 m/s

a) Find the acceleration of the proton

b) How long does it take the proton to reach the speed?

c) How far has it moved in this time?

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